18U501 (Pages: 3) Name: ………………………

Reg. No……………………..

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMEBR 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT5 B05/CC18U MAT5 B05 - VECTOR CALCULUS

(Mathematics - Core Course)

(2015-Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. Find domain of the function $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$
2. Evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{e^{\sqrt{y}} sinx}{x}$
3. Find the first order partial derivative of $f(x)$ with respect to x at the point $(2,0)$, where $f(x,y) = 9x^2y + 8xy^2 - 12xy + 3y - 5$
4. What is $\frac{dw}{dt}$ when $w = sinh^{-1}x$, where $x = t^3$
5. State Taylors formula.
6. If $f(x,y,z) = 4x^2yz - y^3z^2$, find ∇f
7. Find an equation for the tangent plane to the surface $xy^2 - 2xz + yz^2 = 1$ at the point $(-1, 2, 0)$.
8. Evaluate $\int_1^2 \int_0^1 (x^2 + y) dxdy$.
9. Find the Jacobian of the transformation $x = r\cos\theta$, $y = r\sin\theta$
10. Give a parametrization of the cylinder $(x-2)^2 + y^2 = 16$, $0 \le z \le 6$.
11. Find the curl of $\mathbf{F}(x,y) = (3x^2 - 2y)\boldsymbol{i} + (2xy - y^{2)}\boldsymbol{j}$.
12. Check whether $\mathbf{F} = yz\boldsymbol{i} + xz\boldsymbol{j} + xy\boldsymbol{k}$ is solenoidal.

(12 x 1 = 12 Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. The plane $x = 2$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(2, 3, 13)$.
14. Integrate $f(x,y,z) = x - 3y^2 + z$ over the line segment C joining the origin and the point $(1, 1, 1)$.
15. If $z = tan^{-1}\frac{y}{x}$, find $\frac{\partial^2 z}{\partial x^2}$.

16. What is the linearization $L(x,y)$ of the function $f(x,y) = x^2 + y^2 + 1$ at $(1,1)$.

17. If $w = x^2 + y - z + sint$ and $x + y = t$, find $(\frac{\partial w}{\partial x})_{y,z}$

18. Find the direction in which f increases most rapidly at the point (1,1,0), if $f(x,y,z) = e^{xy}cosz$.

19. Find the local extreme values of the function $f(x,y) = x^2 + xy + 3x + 2y + 5$.

20. Using Taylor's formula for $f(x,y)$ at the origin, find a cubic approximation of $f(x,y) = xe^y$ near the origin.

21. Find the area enclosed between $x = 2,\ x = 8$ and $y = x\ ,\ y = x - 1$.

22. Evaluate the cylinder co-ordinate integral $\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} dzrdrd\theta$.

23. Calculate the area enclosed by the lemniscate $r^2 = 4cos2\theta$.

24. Calculate the work done by $\mathbf{F} = (x-y)\boldsymbol{i} + (y-z)\boldsymbol{j} + (z-x)\boldsymbol{k}$ over the curve $\boldsymbol{r}(t) = t\boldsymbol{i} + 2t\boldsymbol{j} + 3t\boldsymbol{k};\ 0 \le t \le 1$ from $(0,0,0)$ to $(1,2,3)$.

25. Evaluate the circulation of the field $\mathbf{F} = (x-y)\boldsymbol{i} + x\boldsymbol{j}$ around the circle $\boldsymbol{r}(t) = cost\boldsymbol{i} + sint\boldsymbol{j},\ 0 \le t \le 2\pi$.

26. Show that $\mathbf{F} = (2x - 3y)\boldsymbol{i} + 2zx\boldsymbol{j} + sinz\boldsymbol{k}$ is not conservative.

(10 x 4 = 40 Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. Using two path test, show that $f(x,y) = \frac{4x^7y^3}{x^{14}+y^6}$ has no limit as (x,y) approaches $(0,0)$.

28. If $v = \frac{1}{r}$ and $r^2 = x^2 + y^2$, show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{1}{r^3}$.

29. Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

30. Evaluate $\iint xydxdy$ over the first quadrant of the circle $x^2 + y^2 = a^2$.

31. Find the volume of the upper region D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$.

32. Calculate the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

33. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2,\ z \ge 0$ by the cylinder $x^2 + y^2 = 1$.

34. Using Stoke's theorem, evaluate $\oint \mathbf{F}.d\boldsymbol{r}$ for the hemisphere $S: x^2 + y^2 + z^2 = 9,\ z \ge 0$, its bounding circle $C: x^2 + y^2 = 9,\ z = 0$ and the field $\mathbf{F} = y\boldsymbol{i} - x\boldsymbol{j}$.

35. Verify that $\iint_S \mathbf{F}.\boldsymbol{n}\,d\sigma = \iiint_D \nabla.\mathbf{F}\,dV$ for the field $\mathbf{F} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

(6 x 7 = 42 Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. Change the order of integration and evaluate the double integral $\int_0^1\int_x^{2-x} \frac{x}{y}\,dydx$.

37. Show that $2xydx + (x^2 - z^2)dy - 2yzdz$ is exact. Hence find the value of the integral $\int_{(0,0,0)}^{(1,2,3)} 2xydx + (x^2 - z^2)dy - 2yzdz$.

38. Write the normal form and tangential form of Green's theorem.
Verify the tangential form of Green's theorem if $M = xy$ and $N = x^2$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

(2 x 13 = 26 Marks)

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