18U502 (Pages: 2) Name: ………………………

Reg. No……………………..

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMEBR 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT5 B06/CC18U MAT5 B06 - ABSTRACT ALGEBRA

(Mathematics - Core Course)

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. Order of an identity element of a group is …………
2. Give an example of a finite non abelian group.
3. Define a permutation on a set.
4. Number of elements in the group A_4 is …………
5. Define index of a subgroup H of G.
6. State TRUE/FALSE: f: GL(n, R) → <R, • > defined by f(A)= det(A) is a homomorphism.
7. State TRUE/FALSE: $\{\rho_0, \rho_1, \rho_{2,}\}$ is a subgroup of S_3.
8. Give an example of a ring which is not a field.
9. Characteristic of ring Z_6 is …………
10. What are the units of the ring < Z, +, •>
11. Define zero divisors of a ring.
12. Compute (12)(3) in Z_{18}.

(12 x 1 = 12 Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Define a group. Is the set of natural numbers a group under addition?
14. Show that {1, -1, i, -i} form a group under multiplication.
15. Show that identity element in a group is unique.
16. Let G be a group then prove that $(a^{-1})^{-1} = a$ for all a ∈ G.
17. Describe Klein 4-group.
18. Show that if $x * x = e$ for all x in a group G, then G abelian.
19. Let G be a group and let $a \in G$. Then prove that $H = \{a^n : n \in Z\}$ is a subgroup of G.

20. Describe all the elements in the cyclic subgroup generated by $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ of GL(2, R)

21. Define Cosets. What are the cosets of 4Z in Z?

22. A group homomorphism $\phi: G \to G'$ is one to one if and only if $Ker(\phi) = \{e\}$

23. Show that identity element is preserved under a group homomorphism.

24. Define a ring. Give an example.

25. Define Kernel of a homomorphism. What is the kernel of the natural homomorphism from Z to Z_4?

26. Show that in a ring $R, a \cdot 0 = 0 \cdot a = 0$ for all a in R

(10 x 4 = 40 Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. Show that subgroup of a cyclic group is cyclic.

28. Show that a finite cyclic group of order n is isomorphic to Z_n.

29. If H & K are subgroups of a group G, show that H∩K is a subgroup of G. Is HUK is a subgroup? Justify.

30. In the ring $\mathbb{Z}_n$ the division of zero are precisely those nonzero elements that are not relatively prime to n.

31. Draw the lattice diagram of Z_{18}.

32. Show that the collection of all permutations S_A on a non empty set A is a group under permutation multiplication.

33. Show that the set of all even permutations in S_n is a group.

34. Let $\varphi: G \to G'$ be a group homomorphism. Show that if $H \leq G$ then $\varphi[H] \leq G'$

35. Show that the characteristics of an integral domain D must be either 0 or a prime p.

(6 x 7 = 42 Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. Show that symmetries of a triangle form a group. Draw it's subgroup diagram.

37. (a) State and prove Lagrange's theorem.

    (b) Show that every group of prime order is Cyclic.

    (c) Show that order of an element of a finite group divides the order of the group.

38. (a) Show that every field is an integral domain.

    (b) Show that every finite integral domain is a field.

(2 x 13 = 26 Marks)

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