18U503 (Pages: 3) Name: ..………………………

Reg. No………………………

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMEBR 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT5 B07/CC18U MAT5 B07 - BASIC MATHEMATICAL ANALYSIS

(Mathematics - Core Course)

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. Determine the set $\{x \in \mathbb{N}: x^2 + 3x - 4 = 0\}$
2. If $A,\ B,\ C$ are sets, then prove that $(A\backslash B) \cap (A\backslash C) \subseteq A\backslash(B \cup C)$
3. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$ and let $E = \{x : 0 \leq x \leq 1\}$. Find $f^{-1}(f(E))$
4. If $a \in \mathbb{R}$, then show that $a.0 = 0$
5. Find the ϵ- neighborhood of a, where $\epsilon = 1$ and $a = -1$
6. If $\inf S = 2$, then find $\inf(3 + S)$, where $3 + S = \{3 + s : s \in S\}$
7. Find $\lim \left(\frac{2n^2-1}{n^2+1}\right)$
8. Show that 2 is not a limit of the sequence $(1,2,1,2,1,2, \ldots)$
9. Find the total length of the removed intervals of Cantor set.
10. Find the multiplicative inverse of $z = 3 + i$
11. Evaluate Re $\frac{1}{2-i}$
12. Find Arg $(2 + i\sqrt{3})$

$(12 \times 1 = 12$ Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Draw the diagram in the plane of the Cartesian product $A \times B$, if $A = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$ and $B = \{y \in \mathbb{R} : y = 1\ or\ y = 2\}$
14. Prove that the set $2\mathbb{N}$ is denumerable.
15. Determine the set $A = \{x \in \mathbb{R} : |x + 1| < |x - 1|\}$
16. If $a, b \in \mathbb{R}$, then prove that $||a| - |b|| \leq |a - b|$
17. Prove that if $x \in \mathbb{R}$, then there exists $n \in \mathbb{N}$ such that $x < n$
18. If x and y are any real numbers with $x < y$, then prove that there exists an irrational number z such that $x < z < y$

 Turn Over

19. Suppose that A and B are nonempty subsets of $\mathbb{R}$ that satisfy the property $a \leq b$ for all $a \in A$ and all $b \in B$. Then prove that $\sup A \leq \inf B$
20. If $X = (x_1, x_2, \ldots, x_n, \ldots)$ be a sequence of real numbers and let $m \in \mathbb{N}$. Prove that if the m-tail $X_m = (x_{m+1}, x_{m+2}, \ldots, x_{m+n}, \ldots)$ of X converges then X converges.
21. If $0 < b < 1$, then show that $\lim(b^n) = 0$
22. Show $\lim\left((n)^{\frac{1}{n}}\right) = 1$.
23. Show that the Cantor set $\mathbb{F}$ has infinitely many points.
24. Find the equation of the circle with center z_0 and radius r.
25. If z_1 and z_2 are any two complex numbers, then prove that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
26. Sketch the region satisfying Re z ≤ 2 and Im z ≥ 2

($10 \times 4 = 40$ Marks)

Section C

Answer any ***six*** questions. Each question carries 7 Marks

27. Let S be a subset of $\mathbb{N}$ that possesses the two properties:

    (i) The number $1 \in S$

    (ii) For every $k \in \mathbb{N}, k \in S$, then $k + 1 \in S$

    Prove that $S = \mathbb{N}$.
28. Let $a, b \in \mathbb{R}$

    (i) If $a.b = 0$, then prove that either $a = 0$ or $b = 0$

    (ii) If $a \neq 0$, and $a.b = 1$, then prove that $b = \frac{1}{a}$
29. Let $S = \{s \in \mathbb{R} : 0 \leq s, s^2 < 2\}$. Prove that $\sup S$ exists and $(\sup S)^2 = 2$
30. Prove that an upper bound u of a nonempty set S in $\mathbb{R}$ is the supremum of S if and only if for every $\epsilon > 0$ there exists an $s \in S$ such that $u - \epsilon < s$.
31. Let (x_n) be a sequence of real numbers that converges to x and let $p(t) = a_k t^k + a_{k-1} t^{k-1} + \cdots + a_1 t + a_0$ be a polynomial in t. Prove the sequence $\left(p(x_n)\right)$ converges to $p(x)$
32. If $X = (x_n)$ is a bounded decreasing sequence, then prove that $\lim(x_n) = \inf\{x_n : n \in \mathbb{N}\}$
33. Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.
34. If the complex numbers z_1, z_2, z_3 are the vertices of an equilateral triangle, prove that
$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1.$$
35. Find the three cube roots of $-8i$.

($6 \times 7 = 42$ Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. (a) Suppose that S and T are sets and $T \subseteq S$. If S is a finite set, then prove that T is a finite set.

    (b) If A is any set, then prove that there is no surjection of A onto the set $\mathcal{P}(A)$ of all subsets of A.
37. (a) If S is a subset of $\mathbb{R}$ that contains at least two points. Suppose S has the property if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$. Prove that S is an interval.

    (b) Prove that $\mathbb{R}$ is uncountable.
38. (a) Prove that if $X = (x_n)$ is a sequence of real numbers, then there is a subsequence of X that is monotone.

    (b) Prove that a Cauchy sequence of real numbers is bounded.

($2 \times 13 = 26$ Marks)

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