18U504 (Pages: 2) Name:……………………….

Reg. No……………………..

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMEBR 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT5 B08/CC18U MAT5 B08 - DIFFERENTIAL EQUATIONS

(Mathematics - Core Course)

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. Determine the order of $\left(\frac{dy}{dx}\right)^2 + y = 0$.
2. Give example of a second order non-homogeneous ordinary differential equation.
3. Find an integrating factor for $(3xy + y^2) + (x^2 + xy)y' = 0$.
4. Determine whether $(2x + 3)dx = (2 - 2y)dy$ is exact or not.
5. Find the Wronskian of x, xe^x.
6. State principle of superposition of second order linear homogeneous differential equations.
7. Use Euler's formula to write e^{1+2i} in the form of $a + ib$.
8. Find $L(t \cosh at)$.
9. Find $L^{-1}\left(\frac{3}{s^2+4}\right)$.
10. Write the general form of Bernoulli Equation.
11. Transform $x_1' = -2x_1 + x_2, x_2' = x_1 - 2x_2$ into a single equation of higher order.
12. Give example of an even function.

(12 x 1 = 12 Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.
14. Show that any separable equation $M(x) + N(y)y' = 0$ is exact.
15. Find the general solution of $y'' + 5y' + 6y = 0$.
16. Solve $y'' + y' + y = 0$.
17. Solve $t^2y'' + ty' + y = 0$ for $t > 0$.
18. Show that convolution integration is commutative.
19. Find $L\big(u_1(t) + 2u_2(t) - 6u_4(t)\big)$.
20. Show that the Laplace transform is a linear operator.

21. Transform $u'' + \frac{1}{2}u' + 2u = 0$ into a system of first order equations.

22. Determine whether $f(x) = \sin\frac{\pi x}{L}$ is periodic. If so, find the fundamental period.

23. True or false: Product of an odd function and an even function is always odd. Justify.

24. Find a_0 in the Fourier series for $f(x) = 3 - x, -3 < x < 3$

25. Use the method of separation of variables to replace the partial differential equation $tU_{xx} + xU_t = 0$ by a pair of ordinary differential equations.

26. Solve the boundary value problem $y'' + 2y = 0, y(0) = 1, y(\pi) = 0$

(10 x 4 = 40 Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. Find the solution of the initial value problem $\frac{dy}{dx} = \frac{y\cos x}{1+2y^2},\ y(0) = 1$

28. Given that $y_1(t) = \frac{1}{t}$ is a solution of $2t^2y'' + 3ty' - y = 0,\ t > 0$. Find a second linearly independent solution.

29. Show that $y_1(t) = e^t,\ y_2(t) = te^t$ form a fundamental set of solution of $y'' - 2y' + y = 0$

30. Solve $y' = t^2y - t, y(0) = 0$ by the method of successive approximation.

31. Find a particular solution of $y'' + 4y = 3\csc t$.

32. Show that $L(\sin at) = \frac{a}{s^2+a^2}, s > 0$

33. Find $L^{-1}\left(\frac{e^{-2s}}{s^2+s-2}\right)$

34. Find Inverse Laplace transform of $F(s) = \frac{1}{s^4(s^2+1)}$

35. State and prove Abel's theorem.

(6 x 7 = 42 Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. Find the solution of $y'' + 2y' + y = 4e^{-t}, y(0) = 2, y'(0) = -1$ using Laplace transform.

37. Find the solution of the heat conduction problem $u_{xx} = 4u_t, 0 < x < 2,\ t > 0$

$$u(0,t) = 0, \qquad u(2,t) = 0$$

$$u(x,0) = 2\sin\frac{\pi x}{2} - \sin\pi x + 4\sin 2\pi x.$$

38. Find the Fourier series of $f(x) = \begin{cases} -x, & -2 \le x < 0 \\ x, & 0 \le x < 2 \end{cases}, f(x+4) = f(x)$.

    Then show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

(2 x 13 = 26 Marks)

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