20P157 (Pages: 2) Name: ……………………..

Reg. No: …………………..

FIRST SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C02 - ANALYTICAL TOOLS FOR STATISTICS – II

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer any ***four*** questions. Weightage 2 for each question

1. If $\alpha_1, \alpha_2, \alpha_3$ are linearly independent in $\mathbb{R}^n$, then prove that the vectors $\alpha_1 + \alpha_2$, $\alpha_2 + \alpha_3$, $\alpha_1 + \alpha_3$ are also linearly independent in $\mathbb{R}^n$.
2. Define rank and signature of a quadratic form with example.
3. If $\bar{A}$ is the g-inverse of A, Show that $A\bar{A}A = \bar{A}$.
4. Find the minimal polynomial of $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$
5. Show that the system $x + 2y = 1, 3x + 6y = 7$ has no solution.
6. State and prove basis theorem.
7. Define an involutory matrix. Show that A is involutory if and only if $(I + A)(I - A) = 0$.

(4 x 2 = 8 Weightage)

PART B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Determine the basis and dimension for the solution space of,

$$x_1 + 2x_2 + 2x_3 - x_4 + 3x_5 = 0$$

$$3x_1 + 6x_2 + 8x_3 + x_4 + 5x_5 = 0$$

$$x_1 + 2x_2 + 3x_3 + x_4 + 5x_5 = 0.$$

9. Classify the quadratic form $3x^2 + 5y^2 + 3z^2 + 2xy + 2yz + 2xz$.
10. Find the singular value decomposition of $A = \begin{bmatrix} 8 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.
11. Find the rank. Also find a basis for row space and column space for $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$.
12. Prove that the geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.
13. Show that any two characteristic vectors corresponding to two distinct characteristic roots of a symmetric matrix are orthogonal.

14. Obtain the g – inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 4 \\ 2 & 4 & 6 \end{bmatrix}$

(4 x 3 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

15. (a) Define an inner product space. For any vectors α, β in an inner product space V, prove that || α +β || ≤ ||α || + || β ||.

(b) Prove that an orthogonal set of non zero vectors is linearly independent.

16. (a) Define a subspace. Let $W = \{\{x, y, z\}; x^2 = y\}$. Is W a subspace of $\mathbb{R}^3$?

(b) Prove that the intersection of two subspaces of a vector space V is a subspace of V.

17. (a) Define Moore Penrose inverse of a matrix. Show that it is unique.

(b) State and prove Cayley – Hamilton theorem.

18. (a) Let U and W be the vector space over the same field F and let T be linear transformation from V into W. Suppose that V is finite dimensional. Prove that rank (T)+ nullity (T) = dim V.

(b) Find the rank and nullity of the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.

(2 x 5 = 10 Weightage)

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