20P159 (Pages: 2) Name: ………………………

Reg. No……………………..

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C04 – PROBABILITY THEORY

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define σ field. Show that intersection of arbitrary number of σ field is a σ field.
2. What do you mean by probability space? If A_n is a sequence of events and A_n converges to A then show that $P(A_n)$ converges to$P(A)$.
3. State and prove Basic inequality.
4. For any characteristic function φ, show that

   (a) Re(1- φ(u)) ≥ (1/4)

   (b) $|\varphi(u) - \varphi(u+h)|^2 \leq 2\{1 - Re\varphi(h)\}$
5. Define almost sure convergence. If X_n is a sequence of random variables such that $X_n \xrightarrow{a.s} X$ then, show that $X_n \xrightarrow{p} X$
6. State and prove Levi's Continuity theorem.
7. State strong law of large numbers. Examine whether it hold for the sequence $\{X_n\}$, if $P(X_k = 1) = \frac{(1-2^{-k})}{2} = P(X_k = -1)$, $P(X_k = 2^k) = 2^{-(k+1)} = P(X_k = -2^k)$

(4 x 2 = 8 Weightage)

Part B

Answer any ***four*** questions. Each question carries 3 weightage.

8. What do you mean by induced probability space? Write down the induced probability space of a random variable X 'number of heads turns up' when a coin is tossed n times
9. Define distribution function. State and prove its properties.
10. Define expectation of a random variable. If X and Y are two random variables, show that $E(X+Y) = E(X) + E(Y)$
11. Show that convergence in probability implies weak convergence. Is the converse true?
12. State and prove Hally- Bray lemma

13. State and prove necessary and sufficient condition for a sequence of random variables to hold WLLN.
14. Examine whether CLT hold for the sequence of independence random variables $\{X_n\}, n = 1, 2, 3\ldots$ where X_n is Poisson with mean λ

(4 x 3 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

15. State and prove (a) Kolmogrov 0-1 law. (b) Borel 0-1 law.
16. (a) Define convergence in r^{th} mean.

    (b) State and prove monotone convergence theorem
17. State and prove Inversion theorem on characteristic function. Obtain the density function if the characteristic function is $e^{-|u|}$
18. State and prove De-Movire's – Laplace central limit theorem.

(2 x 5 = 10 Weightage)

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