20P101 (Pages: 2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C01 - ALGEBRA-I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Give an example of an isometry of the plane which leaves the X- axis fixed.
2. Show that A_n is normal subgroup of S_n.
3. Compute the factor group $(Z_4 \times Z_6)/\langle(0,2)\rangle$
4. Show that Z/nZ isomorphic to Z_n.
5. Show that Z has no composition series.
6. Show that the center of finite non-trivial p-group is nontrivial.
7. Show that no group of order 20 is simple.
8. Let F be the ring of functions mapping $R\ in\ to\ R$ and let C be the subring of all the constant functions in F. Is C is an ideal of F? Justify.

(8 x 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. a) If m divides the order of the group G, show that G has a subgroup of order m.

   b) Find all subgroups of $Z_2 \times Z_4$ of order 4.

10. a) Prove that M is maximal normal subgroup of G if and only if G/M is simple.

    b) Find the center of $Z_3 \times S_3$.

11. a) Let X be a $G - set$ and let $x \in X$ then show that $|Gx| = (G: G_x)$

    b) If G is finite show that $|Gx|$ is a devisor of $|G|$

UNIT - II

12. a) If G has a composition series and if N is proper normal subgroup of G then show that there exist a composition series containing N

    b) Find a composition series of $Z_4 \times Z_9$ containing $\langle(0\ ,\ 1)\rangle$.

13. a) State and prove third sylow theorem.

    b) Find the order of sylow 3- subgroup of group of order 54

14. a) Show that no group of order p^r for $r > 1$, is simple

b) Show that every group of order 255 is cyclic.

UNIT - III

15. a) State and prove evaluation homomorphism for field theory.

b) Find the $Ker\ \varphi_\pi$ of the evaluation homomorphism $\varphi_\pi: Q[x] \to R$, defined by $\varphi_\pi(a_0 + a_1x + \cdots + a_nx^n) = a_0 + a_1\pi + \cdots + a_n\pi^n$.

16. State and prove division algorithm for $F[x]$, where F is field.

17. Show that the set $End(A)$ of all endomorphisms of an abelian group A forms a ring under homomorphism addition and homomorphism multiplication. And then show that this ring need not be commutative by an example.

(2 x 2 = 4 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) Let H be a subgroup of a group G then the left cosets multiplication is well defined by the equation $(aH)(bH) = (ab)H$ if and only if H is normal subgroup of G.

b) If H is normal subgroup of a group G, show that the left cosets of H form a group under the binary operation $(aH)(bH) = (ab)H$

c) Find the order of $5 + \langle 4 \rangle$ in $Z_{12}/\langle 4 \rangle$

19. a) Let H be a subgroup of a group G and let N be normal subgroup of group G then show that $(HN)/N \cong H/H \cap N$

b) Let $G = Z_{24},\ H = \langle 4 \rangle,\ K = \langle 8 \rangle$

(i) List the cosets in G/H

(ii) List the cosets in G/K

(iii) List the cosets in H/K

(iv) Establish the correspondence between G/H and $(G/K)/(H/K)$ by using third isomorphism theorem.

20. a) Determine all groups of order 8 upto isomorphism

b) Give the addition and multiplication tables for the group algebra $Z_2 \times G$ where $G = \{e, a\}$ is a cyclic group of order 2

21. a) State and prove Eisenstein Criterion

b) Using Eisenstein Criterion prove that the polynomial

$\varphi_p(x) = \frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \cdots + x + 1$ is irreducible over Q for a prime p.

(2 x 5 = 10 Weightage)

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