20P102 (Pages: 2) Name: ……………………

Reg. No……………………

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C02 – LINEAR ALGEBRA

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A (Short Answer questions)

Answer ***all*** questions. Each question carries 1 weightage.

1. Let V be vector space over the field F and α be any vector in V, then prove that $(-1)(\alpha) = -\alpha$.
2. Find the value of k such that the set $S = \{(1,2,3), (2,4,0), (k, 1,1)\}$ is linearly dependent in $\mathbb{R}^3$.
3. Find the range and null space of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by
$$T(x, y) = (x + y, x - y).$$
4. Let $\mathcal{B} = \{(1,0,1), (1,,1,1), (2,2,0)\}$ be a basis for $\mathbb{C}^3$. Find the dual basis of $\mathcal{B}$.
5. Show the similar matrices have the same characteristic polynomial.
6. Find the two eigen values of the projection $P: \mathbb{R}^2 \to \mathbb{R}^2$ given by $P(x, y) = (x, 0)$.
7. Find the orthogonal compliment of $W = \{(x, x); x\epsilon\ \mathbb{R}\}$ in $\mathbb{R}^2$.
8. Let $W = Span(1, 2, 1)$ and $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x + z, 2y, 2z)$. Verify whether W is an invariant subspace of T.

(8 x 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Prove that in a finite dimensional vector space, every nonempty linearly independent set of vectors is a part of a basis.
10. Let $B = \{\alpha_1, \alpha_{2,} \alpha_3\}$ be the ordered basis for $\mathbb{R}^3$ consisting of $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1)$ and $\alpha_3 = (1, 0, 0)$.What are the co-ordinates of a vector (a, b, c) in the ordered basis B
11. Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

UNIT II

12. Find the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$ in the ordered basis $\{\alpha_1, \alpha_{2,} \alpha_3\}$ where $\alpha_1 = (1, 0, 1), \alpha_2 = (-1, 2, 1)$ and and $\alpha_3 = (2, 1, 1)$.
13. Define transpose of a linear transformation from a vector space V into the vector space W defined over the field F. Also prove that if V and W are finite dimensional, then range of T^t is the annihilator of the null space of T.
14. Let T be a diagonalizable linear operator and $c_{1,} c_2, \ldots c_k$ be the distinct characteristic value of T. Prove that the characteristic polynomial of T is equal to $(x - c_1)^{d_1}(x - c_2)^{d_2} \ldots (x - c_k)^{d_k}$ for some positive integers $d_{1,} d_{2,} \ldots d_k$.

UNIT III

15. Let $W_{1,}$ W_2 be subspace of a vector space V and let $= W_1 \oplus W_2$. Show that there is a projection on V with range space W_1 and null space W_2.
16. (i) Let (|) be the standard inner product on $\mathbb{R}^2$. Let $\alpha = (1, 2), \beta = (-1, 1)$. If γ is a vector in $\mathbb{R}^2$ such that $(\alpha \mid \gamma) = -1$ and $(\beta \mid \gamma) = 3$ Find γ

    (ii) Show that if S is any subset of a vector space V then its orthogonal complement $S^\perp$ is a subspace of V.
17. State and prove Cauchy-Schwartz inequality.

(6 x 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) Describe all subspace of $\mathbb{R}^3$.

    (b) If A is an $m \times n$ matrix with entries in the field F, then prove that row $rank$ (A) = column rank (A).
19. Let V be an m-dimensional vector space over the field F and W be an n-dimensional vector space over the field F. Then with the usual assumption prove that the space $L(V, W)$ is a finite dimensional vector space of dimension mn.
20. (i) Verify whether the operator given by $T(x, y) = (2x + y, y)$ is diagonalizable.

    (ii) Let W be a subspace of a finite dimensional vector space V. Then show that $dim\, W + dim\, W^0 = dim\, V$
21. State and prove Cayley Hamilton theorem for a linear operator on a finite dimensional vector space.

(2 x 5 = 10 Weightage)

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