20P103 (Pages: 2) Name: ....

Reg. No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1C03 - REAL ANALYSIS-I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries **1** weightage.

1. Prove that every neighbourhood is an open set.
2. Define Cantor set.
3. If E is an infinite subset of a compact set K, then prove that E has a limit point in K.
4. Define Riemann-stieltjes Integral of a real bounded function f with respect to a monotonically increasing function α over $[a, b]$.
5. Let f be defined on $[a, b]$ and are differentiable at a point $x \in [a, b]$, then prove that f is continuous at x.
6. Let f be defined for all real x, and suppose that $|f(x) - f(y)| \leq (x-y)^2$ for all real x and y. Prove that f is a constant.
7. If P^* is a refinement of P, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(P, f, \alpha)$.
8. Define an uncountable set. Prove that the set of all integers is countable.

(8 x 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Define limit point of a set. Construct a bounded set of real numbers with exactly three limit points.
10. Let A be the set of all sequences whose elements are the digits 0 and 1. Then prove that this set A is uncountable.(The elements of A are sequences like $1, 0, 0, 1, 1, ...$)
11. Let p be a limit point of a set E in a metric space, then prove that every neighbourhood of p contains infinitely many points of E.

UNIT II

12. Prove that if f is continuous on $[a, b]$ then $f \in \Re(\alpha)$ on $[a, b]$.
13. If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, then prove that $f \in \Re(\alpha)$.
14. Suppose f is continuous on $[a, b]$, $f'(x)$ exists at some point $x \in [a, b]$, g is defined on an interval I which contains the range of f, and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))$; $(a \leq t \leq b)$, then h is differentiable at the point $f(x)$, and $h'(x) = g'(f(x))f'(x)$.

UNIT III

15. Let $f \in \Re(\alpha)$ on $[a,b]$, and for $a \leq x \leq b$, let $F(x) = \int_a^b f(t)dt$. Prove that F is continuous on $[a,b]$.

16. Show by an example that the limit of the integral need not be equal to the integral of the limit, even if both are finite.

17. If $\{f_n\}$ is a point wise bounded sequence of complex functions on a countable set E, then show that $\{f_n\}$ has a subsequence $\{f_{n_k}(x)\}$ converges for every $x \in E$.

(6 x 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
    (b) Show that a continuous function maps a connected subset of metric space onto a connected set.
    (c) If f is a continuous mapping of a compact metric space X into R^k , then prove that $f(X)$ is closed and bounded. Thus, f is bounded

19. (a) Prove that finite set has no limit point.
    (b) Prove that compact subset of a metric space is closed.
    (c) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(X)$ is compact

20. (a) Suppose $f \in \Re(\alpha)$ on $[a,b]$, $m \leq t \leq M$, ϕ is continuous on $[m,M]$, and $h(x) = \phi(f(x))$ on $[a,b]$. Then prove that $h \in \Re(\alpha)$ on $[a,b]$.
    (b) Assume α increases monotonically and $\alpha^{'} \in \Re$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Then prove that $f \in \Re(\alpha)$ if and only if $f\alpha^{'} \in \Re$. In that case Integral $\int_a^b f d\alpha = \int_a^b f(x)\alpha^{'}(x)dx$

21. State and prove Stone-Weierstrass theorem.

(2 x 5 = 10 Weightage)

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