20P156 (Pages: 2) Name: ………….………….

Reg. No…………..………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P STA1 C01 – ANALYTICAL TOOLS FOR STATISTICS – I

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define the following (a) Directional derivative (b) Total derivative of a multivariable function (c) Riemann integral of a multivariable function.
2. Define limit of a multivariable function and find $\lim_{(x,y)\to(0,0)} \frac{\sin\,(x^2+y^2)}{(x^2+y^2)}$.
3. Define an analytic function and find the analytic function whose real part is $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.
4. State Cauchy's theorem for an analytic function and evaluate $\int_1^3 \frac{dz}{(z-4)z}$.
5. Evaluate $\int_0^{2\pi} \frac{d\theta}{1+a\sin\theta}$ *where* $-1 < a < 1$..
6. State and prove Polar form of Cauchy Riemann equation.
7. Define Laplace transform of a function. Obtain the same of a constant function.

(4 x 2 = 8 Weightage)

PART B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Define local maxima and minima of a multivariable function. Examine the function $x^3 + y^3 - 3x - 12y + 20$ for maxima and minima.
9. State and prove Taylors theorem.
10. Explain the different types of singularities and give examples for each.
11. Solve the differential equation $(D^2 + D)x = 2$, when $x(0) = 3$ and $x'(0) = 1$ where $Dx = \frac{dx}{dt}$
12. State residue theorem. Integrate $\int_c \frac{e^{3z}}{(z^2-2)(z^2-6z+5)}\, dz$ where c is the circle $|z| = 4$ using residue theorem
13. Find the Fourier transform of $f(x) = x \quad if \quad a \le x \le b$..

14. Determine the inverse Laplace transform of (a) $(1 + te^{-t})^2$ (b) $5e^{-2t} + 2e^{-3t}$.

(4 x 3 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

15. (a) Explain the method of Lagangian multiplier method of finding the optima.

(b) Find the points on the sphere $x^2 + y^2 + z^2 = 36$, that are closest and farthest from the point (1,2,2).

16. State and prove the necessary and sufficient condition for a function to be analytic.

17. (a) State and prove Laurent's theorem.

(b) Find Laurents series expansion of $\frac{4}{(z-1)(z+1)}$, specifying the regions.

18. State and prove Poisson's Integral formula.

(2 x 5 = 10 Weightage)

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