20P158 (Pages: 2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P STA1 C03 – DISTRIBUTION THEORY

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define Hyper geometric distribution. Find its mean
2. What is meant by Lognormal distribution? How it is related to Normal distribution
3. Give any one distribution for which moment generating function does not exist but possesses additive property
4. What is mean by mixture distribution? Give an example
5. If X has uniform distribution in $(0, 1)$. Find the distribution of $-2logX.$
6. Find the mgf of Geometric distribution. Hence find its r^{th} raw moment
7. Define r^{th} order statistic and obtain its pdf

(4 x 2 = 8 Weightage)

Part B

Answer any ***four*** questions. Each question carries 3 weightage.

8. State the relationship between Normal, Chi-square, t and F distributions
9. Show that the first order statistic arising from a random sample of size n from $U(0,1)$ distribution has Beta distribution
10. If $Y = [Y_1, Y_2 \dots . Y_n]'$ is a vector of random variables with probability density function $f(y_1, y_2, \dots, y_n) = k e^{-\frac{1}{2}[(Y-\theta)'\Sigma(Y-\theta)]}$, $-\infty < y < \infty$ and Σ is a $n\ x\ n$ positive definite matrix then prove that (i) $k = \frac{1}{(2\pi)^{\frac{n}{2}}} |\Sigma|^{\frac{-1}{2}}$ (ii) $E[y] = \theta, D(y) = \Sigma$
11. Let X and Y be independent random variables following the negative binomial distributions $NB(r_1, P)$ and $NB(r_2, P)$ respectively. Show that the conditional probability mass function of X given $X + Y = t$ is hypergeometric.
12. Let X_1 , X_2 , …. X_n be random sample from Weibull distribution. Obtain the distribution of min $(X_1, X_2 , \dots . X_n)$ and identify its distribution

13. Define probability generating function. If $P(s)$ is the probability generating function associated with a non-negative integer valued random variable show that

$$\sum_{n=0}^{\infty} P(x \leq n) = \frac{p(s)}{1-s}$$

14. If X and Y are independent Gamma variates with parameters (α_1, β_1) and (α_2, β_2) respectively. Show that the variables $U = X + Y$ and $V = \frac{X}{(X+Y)}$are independently distributed. Also identify their distributions

(4 x 3 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Let X_1 and X_2 be two independent standard Cauchy random variables. Find the pdf of
    (a) $Y_1 = \frac{X_1}{X_2}$ (b) $Y_2 = X_1 + X_2$ (c) $Y_3 = X_1^2$
16. Define Power Series family. Obtain the mgf of the distribution and derive the mean and variance from it. Also obtain the recurrence relation satisfied by the cumulants
17. Define non-central t and derive its pdf. Obtain the pdf when the non-centrality parameter become zero.
18. Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ denote the order statistics in a sample of size n from a population with pdf $f(x) = \beta e^{-\beta x}; x > 0, \beta > 0$ and 0 elsewhere. Show that $x_{(r)}, x_{(s)} - x_{(r)}$ are independent for $s > r$.

(2 x 5 = 10 Weightage)

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