19P202 (Pages: 2) Name…………………….

Reg.No………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS - PG)

(Mathematics)

CC19P MTH2 C07 – REAL ANALYSIS II

(2019 Admissions: Regular)

Time: Three Hours Maximum:30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that Lebesgue measure is countably additive.
2. Prove that if a $\sigma -$ algebra of subsets of $\mathbb{R}$ contains intervals of the form(a, ∞), then it contains all intervals.
3. Let g be a measurable real-valued function defined on E and f a continuous real-valued function defined on $\mathbb{R}$. Prove that $f \circ g$ is a measurable function on E.
4. Let f be a bounded measurable function on a set of finite measure E. Then prove that f is integrable over E.
5. Let f be integrable over E and C a measurable subset of E. Show that $\int_C f = \int_E f\chi_C$.
6. Let f be integrable over E. Prove that for each $\varepsilon > 0$, there is a set E_0 with $m(E_0) < \infty$ for which $\int_{E - E_0} |f| < \varepsilon$.
7. Prove that every Lipschitz function defined on a closed, bounded interval [a, b] is absolutely continuous on [a, b].
8. State and prove Chordal Slope Lemma.

(8 x 1 = 8 Weightage)

PART B

Answer any ***two*** from each unit . Each question carries 2 weightage

Unit -1

9. Show that the union of a countable collection of measurable sets is measurable.
10. Prove that any set E of real numbers with positive outer measure contains a subset that fails to be measurable.
11. State and prove Lusin's Theorem.

Unit- 2

12. Let f be a bounded measurable function on a set of finite measure E. Prove that $\int_{A\cup B} f = \int_A f + \int_B f$ where A and B are disjoint measurable subsets of E.
13. Define convergence in measure.
    If $\{f_n\} \to f$ in measure on E, then prove that there is a subsequence $\{f_{n_k}\}$ that converges point wise a. e. on E to f.
14. Let f be a bounded function on a set of finite measure E, then prove that f is Lebesgue integrable over E if and only if it is measurable.

Unit- 3

15. Let the function f be monotone on [a, b]. Prove that f is absolutely continuous on [a, b] if and only if $\int_a^b f' = f(b) - f(a)$.
16. State and prove Jordan's Theorem.
17. Let E be a measurable set and $1 \le p \le \infty$. Suppose $\{f_n\}$ is a sequence in $L^p(E)$ that converges pointwise a.e. on E to the function f which belongs to $L^p(E)$. Prove that $\{f_n\} \longrightarrow f$ in $L^p(E)$ if and only if $\lim_{n\to\infty} |f_n|^p = \int_E |f|^p$.

(6 x 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Prove that the outer measure of an interval is its length. Hence prove that [0, 1] is uncountable.
19. State and prove Vitali Convergence Theorem.
20. If the function f is monotone on the open interval (a, b), then prove that it is differentiable almost everywhere on (a, b).
21. Prove that $L^p(E)$ is a normed linear space for $1 \le p \le \infty$.

(2 x 5 = 10 Weightage)

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