20U314S (Pages: 2) Name: …..…………..……….

Reg. No: ………….…..……….

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

CC15U MAT3 C03 / CC18U MAT3 C03 - MATHEMATICS - III

(Mathematics – Complementary Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

Part-A

Answer ***all*** questions. Each question carries 1 mark.

1. Write the general form of Bernoulli's differential equation.
2. What is the order of the differential equation $y'' + (8x + 3)y' - y = 0$.
3. Find the solution of the differential equation $y' = ky$.
4. Define Singular matrix.
5. The rank of a Zero matrix is ……
6. The eigen values of $\begin{bmatrix} 7 & 0 \\ 0 & 5 \end{bmatrix}$ are ……
7. The work done by a constant force F in making a displacement d is given by ……
8. Define Irrotational vector.
9. State Laplace's equation.
10. If $f = x^2 + y^2 + z^2$, find grad f.
11. Find the velocity of a particle with position vector $r(t) = sint\ \boldsymbol{i} + t\ \boldsymbol{j} + 1\ \boldsymbol{k}$.
12. What is the volume of the parallelopiped with edge vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$.

(12 × 1 = 12 Marks)

Part B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Show that $(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2)dy = 0$ is exact.
14. Find the integrating factor of $xy\ dx + (2x^2 + 3y^2 - 20)dy = 0$.
15. Solve the initial value problem $y' = -\frac{y}{x}$; $y(1) = 1$.
16. Find the characteristic roots of $\begin{bmatrix} -3 & 2 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$
17. Show that $x + 2y = 3$, $2x + 4y = 7$ is consistent.
18. A force $F = 1i + 1j + 1k$ acts through a point $A(-2, 3, 1)$. Find the moment vector m of F about a point $Q(1, 2, 3)$.
19. Are the vectors $[1, 2, 1]$, $[3, 2, -7]$, $[5, 6, -5]$ linearly independent?

20. Find the unit tangent vector to the curve $x = t, y = t^2, z = t^3$ at the point (2,4,8).

21. Find the divergence of $\boldsymbol{v} = xyz\,\boldsymbol{i} + 3x^2y\,\boldsymbol{j} + (xz^2 - y^2z)\,\boldsymbol{k}$ at $(1, 2, -1)$.

22. Write the parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$ with center $(0, 0, 0)$ and radius $\boldsymbol{a}$.

23. Define Jacobian.

24. State Gauss's Divergence Theorem.

(9 × 2 = 18 Marks)

Part C

Answer any ***six*** questions. Each question carries 5 marks.

25. Find the orthogonal trajectories of the family of parabolas $y = cx^2$.

26. Solve $\frac{dy}{dx} + x\,sin2y = x^3cos^2y$.

27. Reduce the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ to its normal form.

28. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, find A^2 using Cayley Hamilton theorem.

29. Find the length of the Catenary $r(t) = t\,i + \cosh t\,j$ from $t = 0$ to $t = 1$.

30. Evaluate the integral $I = \int_C 3x^2dx + 2yzdy + y^2dz$ from $A: (0, 1, 2)$ to $B: (1, -1, 7)$ by showing that F has a potential.

31. The coordinates of a particle at time t are $x = sint - t\,cost,\ y = cost + t\,sint,\ z = t^2$. Find the speed, the normal and tangential components of acceleration.

32. Find the area enclosed by the cardioid $r = a(1 - cos\theta)$.

33. Find the volume of the region between the cylinder $z = y^2$ and the xy −plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$.

(6 × 5 = 30 Marks)

Part D

Answer any ***two*** questions. Each question carries 10 marks.

34. Solve using Cramer's rule $2x - y + 3z = 9$, $x + y + z = 6,\ x - y + z = 2$.

35. Solve the initial value problem $2xy\frac{dy}{dx} - y^2 + x^2 = 0,\ \ y(1) = 1$.

36. Verify Green's theorem in the plane for $\oint_C (xydx + x^2dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

(2 × 10 = 20 Marks)

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