20U300 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS3 C03 - MATHEMATICS - III

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find the vector function that describes the curve C of intersection of the plane \(y=2x\) and the paraboloid \(z=9-x^2-y^2\).
2. If \(z=4x^3y^2-4x^2+y^6+1\), find \(\dfrac{\partial z}{\partial x}\)
3. Find the level curve of \(f(x,y)=\frac{x^2}{4}+\frac{y^2}{9}\) passing through the point \((-2,-3)\)
4. If \(\vec{r}=x \vec{i}+ y\vec{j}+z\vec{k} \), prove that \(\nabla \times \vec{r} =0 \)
5. Show that the line integral \(\displaystyle \int_{(0,0)}^{(2,8)} (y^3+3x^2y)dx+(x^3+3y^2x+1)dy \) is path independent.
6. State Stokes' theorem.
7. Convert the equation \(x^2+z^2=16 \) to cylindrical coordinates.
8. State the divergence theorem.
9. Express the complex number \(i(5+7i)\) in the form \(a+ib\).
10. Show that the function \(f(z) =x + 4iy\) is nowhere differentiable.
11. Express \(e^{-{{\pi}\over 3}i}\) in the form \(a +ib\).
12. State Cauchy-Goursat theorem.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. If \(\textbf{r}(t) =t \textbf{i}+\frac{1}{2} t^2 \textbf{j}+\frac{1}{3}t^3\textbf{k} \) gives the position vector of a moving particle. Find the tangential and normal components of acceleration at any time t. Find the curvature.

14. Find the directional derivative of $f(x,y)=2x^2y^3+6xy$ at $(1,1)$ in the direction of a unit vector whose angle with the positive x-axis is $\frac{\pi}{6}$

15. Evaluate $\displaystyle \int_Cxydx+x^2dy$ where C given by $y=x^3 ; -1\leq x\leq 2.$

16. Evaluate $\displaystyle \iint_Rxe^{y^2}dA$ over the region R in the first quadrant bounded by the graphs of $y=x^2, \quad x=0,\quad y=4.$

17. If T is the transformation from spherical to rectangular coordinates, show that $\dfrac{\partial (x,y,z)}{\partial (\rho, \phi, \theta}=\rho^2 \sin \phi.$

18. Verify that the function $u(x,y) =x^2-y^2$ is harmonic. Also find *v*, the harmonic conjugate of *u*.

19. Evaluate $\displaystyle\int_{C}\frac{1+z}{z}\; dz$, where C is the right half of the circle $|z|=1$, $z=-i$ to $z=i$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. Verify Green's theorem by evaluating both the integrals, $\displaystyle \oint_C \left((x-y)dx +xy dy\right)=\iint_R (y+1)dA$ where C is the triangle with vertices $(0,0), (1,0),(1,3)$ taken in anticlockwise direction.

21. State Cauchy's integral formula. Using it evaluate $\displaystyle\oint_{C}\frac{e^{z^{2}}}{z-i^{}}dz$, where C is the circle $|z-i|=1.$

(1 × 10 = 10 Marks)

****