19U501S (Pages: 3) Name: ……………..………

Reg. No: ………………..

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

CC15U MAT5 B05/CC18U MAT5 B05 - VECTOR CALCULUS

(Mathematics – Core Course)

(2015 to 2018 Admissions - Supplementary/ Improvement)

Time: Three Hours Maximum: 120 Marks

SECTION A (Objective type)

Answer ***all*** questions - Each question carries 1 mark.

1. Graph the level curve $f(x,y) = 75$ of the function $f(x,y) = 100 - x^2 - y^2$.
2. Evaluate $\lim_{(x,y,z)\to\left(\frac{\pi}{2},0,2\right)} ze^{-2y}\cos 2x$.
3. Find $\frac{\partial f}{\partial x}$, if $f(x,y) = \sin(2x - 3y)$.
4. Write 2-dimensional Laplace equation.
5. Find the linearization of $f(x,y) = x^2 - y^2$ at $(1,1)$.
6. Evaluate $\int_0^1\int_0^1 (x+y)\,dx\,dy$.
7. Find the limits of the double integral $\iint_R f(x,y)\,dA$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.
8. Find the Jacobian in changing a double integral from Cartesian coordinates into polar coordinates.
9. What is the volume of a closed bounded region D in space?
10. Find gradient field of the function $g(x,y,z) = e^z - x^2y^3$.
11. Write a potential function for the conservative field $\boldsymbol{F} = 2x\,\boldsymbol{i} + 2y\,\boldsymbol{j} + 4z\,\boldsymbol{k}$.
12. Show that the field $\boldsymbol{F} = (z+y)\boldsymbol{i} + z\boldsymbol{j} + (y+x)\boldsymbol{k}$ is not conservative.

(12 × 1 = 12 Marks)

Part B (Short answer type)

Answer any ***ten*** questions. Each question carries 4 marks.

13. Find the domain and range of the function $f(x,y) = \frac{1}{\sqrt{4-x^2-y^2}}$.
14. Show that the function $f(x,y) = \frac{x}{\sqrt{x^2+y^2}}$ have no limit as $(x,y) \to (0,0)$.
15. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the function $f(x,y) = tan^{-1}\left(\frac{y}{x}\right)$.
16. Show that the function $w = \cos(2x + 2ct)$ satisfies the wave equation $\frac{\partial^2 w}{\partial t^2} = c^2\frac{\partial^2 w}{\partial x^2}$.

17. Find the derivative of $f(x,y) = 2xy - 3y^2$ at $P_0(5,5)$ in the direction of the vector $4i + 3j$.
18. Find an equation for the tangent line to the circle $x^2 + y^2 = 4$ at the point $(0,-2)$.
19. Evaluate $\iint_R xy\,dA$, where R is the region $x^2 + y^2 \le 25, x \ge 0, y \ge 0$.
20. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$.
21. Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x, x = 0$ and $x + y = 2$ in the xy −plane.
22. Find the area of the region cut from the first quadrant by the cardioid $r = 1 + \sin\theta$.
23. Evaluate $\iint_R e^{x^2+y^2}\,dx\,dy$, where R is the semi-circular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.
24. Find the integral of $f(x,y,z) = x - y + z - 2$ over line segment from $(0,1,1)$ to $(1,0,1)$.
25. Find $\bar{x}$, the x-coordinate of center of mass, of a wire of density $\delta(x,y,z) = 15\sqrt{y+2}$ lies along the curve $r(t) = (t^2 - 1)\,j + 2t\,k, -1 \le t \le 1$.
26. A fluids velocity fluids field is $\boldsymbol{F} = x\,i + z\,j + y\,k$. Find the flow along the helix $r(t) = \cos t\,i + \sin t\,j + t\,k, 0 \le t \le \frac{\pi}{2}$.

(10 × 4 = 40 Marks)

Part C (Short essay type)

Answer any ***six*** questions. Each question carries 7 marks.

27. Show that the function $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is continuous everywhere except at the origin.
28. Find all second order partial derivatives of the function $f(x,y) = xy^2 + \sin(xy) - 100$.
29. Find the linearization of $f(x,y,z) = xy + 2yz - 3xz$ at the point $(1,1,0)$. Also find an upper bound for the magnitude of the error in this approximation over the region $|x-1| \le 0.01$, $|y-1| \le 0.01$, $|z| \le 0.01$.
30. Find the absolute maximum and minimum values of $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0, y = 2, y = 2x$.
31. Find the points on the sphere $x^2 + y^2 + z^2 = 1$ farthest from the point $(2,1,2)$.
32. Using the transformations $u = x - y$ and $v = 2x + y$, evaluate $\iint_R (2x^2 - xy - y^2)dA$, for the region R in the first quadrant bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$ and $y = x + 1$.

33. Find the volume of the tetrahedron with vertices $(0,0,0), (1,1,0), (0,1,0)$ and $(0,1,1)$.
34. Find volume of the "ice cream cone" D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$.
35. Using Green's theorem calculate the area enclosed by the circle $\boldsymbol{r}(t) = \text{a}\cos t\;i + a\sin t\,j, 0 \le t \le 2\pi$.

$(\mathbf{6 \times 7 = 42\ Marks})$

Part D (Essay type)

Answer any **two q**uestions. Each question carries 13 marks.

36. Find the derivative of $f(x,y,z) = \ln(x^2 + y^2 - 1) + y + 6z$ at the point $P_0(1,1,0)$ in the direction of the vector $\vec{\imath} - 3\vec{\jmath} + 4\vec{k}$. In what directions does the function change most rapidly at the given point P_0 and what are the rates of change in these directions.
37. (a) If $\sin z = \frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z$.

    (b) Give a reasonable square centered at (1,1) over which the value of $f(x,y) = x^3y^4$ will not vary by more than ± 0.1
38. Verify both forms of Green's theorem for the field $F = -x^2y\,i + xy^2\,j$ and the region R bounded by the circle $\boldsymbol{r}(t) = \cos t\;i + \sin t\,j, 0 \le t \le 2\pi$.

$(\mathbf{2 \times 13 = 26\ Marks})$

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