19U502S (Pages: 2) Name: ………………………

Reg. No: …………………..

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS- UG)

CC15U MAT5 B06/CC18U MAT5 B06 - ABSTRACT ALGEBRA

(Mathematics – Core Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time : 3 Hours Maximum: 120 Marks

PART - A (Objective type)

Answer ***all*** questions. Each question carries 1 mark.

1. State true /false: If G is a group of order 19 then G is cyclic.
2. The Klein 4-group has how many proper non trivial subgroups?
3. Write the number of cosets of $6\mathbb{Z}$ in $\mathbb{Z}$.
4. Determine whether the function $f(x) = x^2$ is a permutation of $\mathbb{R}$.
5. Write the cycle $(1, 3, 5, 6, 2)$ in S_6 as a product of transpositions.
6. The kernel of the natural map (canonical map) $\gamma : \mathbb{Z} \to \mathbb{Z}_n$ is ....
7. How many unit elements are there in the ring $\mathbb{Z}$.
8. Give an example of an integral domain which is not a field.
9. The characteristic of the ring $\mathbb{R}$ is ....
10. Give an example of a ring with unit element.
11. A non-commutative division ring is called .....
12. The number of divisors of zero in $\mathbb{Z}_6$.

(12 × 1 = 12 Marks)

PART- B (Short Answer Type)

Answer any ***ten*** questions. Each question carries 4 marks.

13. Let G be a group. If the inverse of a is a^{-1},then show that inverse of a^{-1} is a.
14. Prove that every cyclic group is abelian.
15. If a and b are any two elements of a group $\langle G, * \rangle$, then the linear equations $x * a = b$ have unique solution x in G.
16. Find all generators of $\mathbb{Z}_{14}$.
17. Find the orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$.
18. Exhibit the left and right coset of the subgroup $6\mathbb{Z}$ of $\mathbb{Z}$.
19. A homomorphism ϕ of a group G is a one-to-one function iff kernel of ϕ is $\{e\}$.
20. Show that the binary structures $\langle \mathbb{Q}, + \rangle$ and $\langle \mathbb{Z}, + \rangle$ under usual addition are not isomorphic.

21. Describe the Klein-4 group V.

22. Find the index of $\langle 4 \rangle$ in $\mathbb{Z}_{24}$.

23. Prove that every group of prime order is cyclic.

24. If $\phi : G \to G'$ is a homomorphism then prove that $\phi(e) = e'$ and $\phi(a^{-1}) = \phi(a)^{-1}$.

25. Let $\phi : G \to G'$ be a homomorphism of a group G onto a group G'. If G is abelian, then prove that G' is also abelian.

26. Let $\langle R, +, \cdot \rangle$ be a ring with additive identity 0. Then for any $a, b, c \in R$ prove that

    (a) $0 \cdot a = a \cdot 0 = 0$

    (b) $a \cdot (-b) = (-a) \cdot b = -(ab)$.

(10 × 4 = 40 Marks)

PART- C (Short Essay Type)

Answer any ***six*** questions. Each question carries 7 marks.

27. Show that the set $\mathbb{Q}^+$ of all positive rational numbers forms an abelian group under the operation defined by $a * b = \dfrac{ab}{2}$

28. Let $\phi : G \to G'$ be a group homomorphism of a group G on to G' . If G is abelian then G' is abelian.

29. Show that the subgroup of a cyclic group is cyclic.

30. Show that cancellation law hold in a ring R iff R has no divisors 0.

31. Define S_n and show that S_3 is group.

32. Find all subgroups of $\mathbb{Z}_{10}$ and draw its lattice diagram.

33. If H and K are two subgroups of G, prove that $H \cap K$ is a subgroup of G.

34. Find $\sigma^{-1}\tau\sigma$ and σ^{100} for the following permutations σ and τ in S_6.
    $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$

35. Show that cancellation law hold in a ring R if and only if R no zero divisors.

(6 × 7 = 42 Marks)

PART - D (Essay Type)

Answer any *two* questions. Each question carries 13 marks.

36. (a) If G is a group, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$. for all $a, b \in G$.

    (b) State and prove Lagrange's theorem.

37. State and Prove Cayley's theorem.

38. (a) Prove that every field is an intergral domain.

    (b) Show that every finite integral domain is a field.

(2 × 13 = 26 Marks)

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