19U504S (Pages: 2) Name: ….………..……….

Reg. No: ……………….……….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

CC15U MAT5 B08/CC18U MAT5 B08 - DIFFERENTIAL EQUATIONS

(Mathematics – Core Course)

(2015 to 2018 Admissions - Supplementary/Improvement)

Time: Three Hours Maximum: 120 Marks

Part A

Answer ***all*** questions. Each question carries 1 mark.

1. If $y = e^{rt}$ is a solution of $y'' + 2y' - y = 0$. Then find r.
2. State whether the equation $y'' + y^2 t = \sin t$ is linear or non-linear. Why?
3. Write down the general form of Bernoulli's equation
4. Give the general solution of $y'' + by' + cy = 0$ whose characteristic equation has a root $\lambda + i\mu$.
5. Solve $y'' - y = 0$
6. Are the functions $\sin x$ and $\cos x$ linearly independent?
7. Write the initial value problem $2y'' - 5y' + y = 0; y(3) = 6, y'(3) = -1$ as a system of first order initial value problems.
8. Show that $x^{(1)}(t) = \begin{bmatrix} e^{3t} \\ 2e^{3t} \end{bmatrix}$ is a solution of $x' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} x$
9. $\mathcal{L}(e^{-at} \sin bt) =$ …………….
10. $\mathcal{L}^{-1}\left(\frac{s}{(s-2)^2}\right) =$ ……………..
11. Is the function $f(x) = x|x|$ even, odd or neither?
12. What is the heat conduction equation?

$(\mathbf{12 \times 1 = 12\ Marks})$

Part B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Solve the initial value problem $ty' + (t+1)y = 0, y(\ln 2) = 1$.
14. Solve $\frac{dy}{dx} = \frac{ay+b}{cy+d}$, where a, b, c, d are constants.
15. Without solving find an interval in which the differential equation $(t^2 - 9)y' + 2y = \ln(20 - 4t)\, ; y(4) = -3$ has a unique solution.
16. State and prove the principle of superposition.
17. Find the general solution of $y'' + 2y' + 1.25y = 0$.
18. Find a particular solution of $y'' - y - -2y = 6e^t$
19. Using method of reduction of order solve the differential equation $t^2 y'' - 5ty' + 9y = 0; t > 0$ given $y = t^3$ is a solution.

20. Find $f * g$ if $f(t) = t$ and $g(t) = e^t$
21. Find the inverse Laplace transform of $F(s) = \frac{1}{s^2-4s+5}$
22. Define the Dirac delta function and find its Laplace transform.
23. Let $f(x) = x$ where $0 \le x \le 1$. Find the 2-periodic even extension of *f*.
24. Find the half range sine series of the function $f(x) = x$ for $0 \le x \le \pi$
25. Determine whether $\sin 4x$ is periodic. If so find its fundamental period.
26. Using the method of method of separating variables solve $u_x + u_y = 0$

(10 × 4 = 40 Marks)

Part C

Answer any ***six*** questions. Each question carries 7 marks.

27. Find an integrating factor and solve the differential equation
$$(x^2 - 2x + 2y^2)dx + 2xydy = 0$$
28. State and prove Abel's theorem.
29. Show that the initial value problem $y' = y^{1/3}$, $y(0) = 0$ has more than one solution. Does it contradict the existence and uniqueness theorem?
30. Solve the initial value problem $4y'' + 12y' + 9y = 0,\ \ y(0) = 0$ and $y(0) = -4$
31. Find the general solution of the differential equation $x^2y'' - 4xy' + 6y = 21x^{-4}$
32. Find $\mathcal{L}^{-1}\left(\frac{s}{(s+1)(s-2)^3}\right)$
33. Find the inverse Laplace transform of $\ln\left(\frac{s+a}{s+b}\right)$
34. Find the Fourier series of the function $f(x) = \begin{cases} \pi + x & -\pi \le x < 0 \\ \pi - x & 0 \le x < \pi \end{cases}$
35. A string is stretched and fastened to two points L apart. Motion is started by displacing the string into the form $u = k(Lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of *x* from one end at time *t*.

(6 × 7 = 42 Marks)

Part D

Answer any ***two*** questions. Each question carries 13 marks.

36. Find the general solution of $y'' - 3y' - 4y = 3e^{2t} + 2\sin t$
37. Solve the initial value problem $y'' + y = \sin 2t\,, y(0) = 2,\ y'(0) = 1$ using Laplace transform.
38. Find the Fourier series expansion of the function $f(x) = x^2,\ -\pi \le x \le \pi$. Also deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

(2 × 13 = 26 Marks)

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