19U502 (Pages: 2) Name: ……..……….……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-UG)

CC19U MTS5 B06 - BASIC ANALYSIS

(Mathematics – Core Course)

(2019 Admission - Regular)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 4

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. If $A_i = \{i, i+1, i+2, ...\}$ find (i) $\bigcup_{i=1}^{n} A_i$ and (ii) $\bigcap_{i=1}^{n} A_i$.
2. Show by a suitable example: If x and y are irrational then x + y and xy need not be irrational.
3. If $0 \leq a < \varepsilon \ \ \forall \varepsilon > 0$, prove that $a = 0$.
4. Prove that √2 is irrational.
5. Show that inf { $\frac{1}{n}$: n ∈ ℕ}= 0.
6. Prove that there can be only one supremum for a given subset S of ℝ.
7. Let r < 0 be a negative real number. Use the Archimedean property to prove that there is an $n \in N$ such that r < $\frac{-1}{n}$< 0.
8. Can we express null set ∅ as an interval? Justify.
9. Show that {1, 0, 1, 0, …} is not convergent.
10. Show that a convergent sequence of real numbers is bounded.
11. Define modulus of a complex number $x + iy$. Find $|2 + i|$.
12. Evaluate $(1 + 2i)(1 - 2i)$.
13. Find principal argument of $1 - i$
14. Identify the set of points representing $0 < |z| < 1$ in the Complex plane.
15. Find $f(i)$ given $f(z) = z^2 - (2 + i)z$.

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. State and prove Cantor's theorem.
17. Prove that contractive sequence converges in **R**.
18. State and Prove monotone convergence theorem.
19. Prove that $sup(a + S) = a + sup\ S$

20. Using nested interval property, prove the uncountability of **R**

21. Prove that if $J_n = (0, \frac{1}{n})$ for $n \in \mathbb{N}$, then $\bigcap_{n=1}^{\infty} J_n = \phi$.

22. Show that $|z_1 z_2|=|z_1||z_2|$

23. Find $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3$

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. State and Prove nested intervals theorem.

25. State and prove characterization theorem of intervals.

26. Prove that there exists $x \epsilon R$ such that $x^2 = 2$.

27. Find the image of circular arc defined by |z|=2, $0 \le \arg z \le \frac{\pi}{2}$, under the mapping $w = z^2$.

(2 × 10 = 20 Marks)

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