19U505 (Pages: 2) Name: ……..……….……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-UG)

CC19U MTS5 B09 - INTRODUCTION TO GEOMETRY

(Mathematics – Core Course)

(2019 Admission - Regular)

Time: 2 Hours Maximum: 60 Marks

Credit: 3

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Find the foci and directrices of the conic $x^2 - 2y^2 = 1$.
2. Determine the slope of the tangent to the curve in R^2 with parametric equations $x = a\cos t, y = b\sin t$, where $t \in (-\pi, \pi], t \neq 0, \pi$.
3. Write the equation of a non-degenerate conic in polar co-ordinates.
4. State Reflection property of Ellipse.
5. Illustrate isometry.
6. Determine whether or not the transformation $t(x) = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ of R^2 is a Euclidean transformation.
7. Give an example of an affine transformation that is not a parallel projection.
8. State Conjugate Diameters Theorem.
9. State the Converse to Menelaus' Theorem.
10. Determine whether the Points $[1, 2, 3], [1, 1, -2], [2, 1, -9]$ are collinear.
11. Distinguish between Collinearity property and Incidence Property of $\mathbb{RP}^2$.
12. Determine the point of intersection of each of the Lines in $\mathbb{RP}^2$ with equations $x + 6y - 5z = 0$ and $x - 2y + z = 0$.

(Ceiling: 20 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

13. Determine the equations of the tangent and the normal to the parabola with parametric equations $x = 2t^2, y = 4t$ at the point with parameter $t = 3$.
14. State and prove Focal distance property of Hyperbola.
15. Prove that Euclidean-congruence is an equivalence relation.
16. Prove that a parallel projection preserves ratios of lengths along a given straight line.
17. Determine the affine transformation which maps the points $(1, -1), (2, -2)\ and\ (3, -4)$ to the points $(8, 13), (3, 4)\ and\ (0, -1)$, respectively.

18. Prove that every hyperbola is affine-congruent to the rectangular hyperbola with equation $xy = 1$.

19. Determine homogeneous coordinates of the form $[a, b, 1]$ for the Points $[2, -1, 4], [4, 2, 8], [2\pi, -\pi, 4\pi], [200, 100, 400], \left[\frac{-1}{2}, \frac{-1}{4}, -1\right]$, $[6, -9, -12]$. Hence decide which homogeneous coordinates represent the same Points.

(Ceiling: 30 Marks)

Section C

Answer any ***one*** question. The question carries 10 marks.

20. Prove that the conic E with equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ is a hyperbola. Determine its center, and its major and minor axes.

21. State and prove Cevas' Theorem.

(1 × 10 = 10 Marks)

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