20U201 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2021

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS2 B02/CC20U MTS2 B02 - CALCULUS OF SINGLE VARIABLE - I

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. If $f(x) = ax^3 + b$, find a and b if it is known that $f(1) = 1$ and $f(2) = 15$.
2. Find $f(x)$, if $f(x+1) = 2x^2 + 7x + 4$.
3. Evaluate $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$.
4. Define limit of a function at a point (Formal definition).
5. Let the functions f and g are continuous at a. Prove that the function $f + g$ is continuous at a.
6. Find the equation of the tangent line to the graph of the function $f(x) = \sqrt{x-1}$ at the point $(4, \sqrt{3})$.
7. The total cost incurred in operating an oil tanker on an 800*mi* run, traveling at an average speed of v mph, is estimated to be $C(v) = \frac{1,000,000}{v} + 200v^2$ dollars. Find the approximate change in the total operating cost if the average speed is increased from 10 mph to 10.5 mph
8. Find the linearization of $f(x) = \sin x$ at $a = \frac{\pi}{4}$
9. Define the critical number of a function.
10. Using Mean value theorem find c if the function $f(x) = x^3 - 2x^2$ is given in $[-1, 2]$
11. Define increasing function with an example.
12. Define vertical asymptote.
13. A car moving along a straight road with velocity function $V(t) = t^2 + t - 6; 0 \le t \le 10$. Where $V(t)$ is measured in feet per second. Find the displacement of the car between $t = 1$ and $t = 4$.
14. What do you mean by solid of revolution. Explain with an example
15. Find the work done by the force $F(x) = 3x^2 + x$ (measured in pounds) in moving a particle along the x-axis from $x = 2$ to $x = 4$ (measured in feets)

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the slope and equation of the tangent to the graph of $f(x) = \frac{1}{x+1}$ at the point $(1, \frac{1}{2})$.
17. Let $s(t) = \frac{2t}{t^2+1}$ be the position of a body moving along a coordinate line. Find the position, velocity and accelaration of the body at $t = 2$.

18. The betty moore company requires that its garbage containers have a capacity of 64 cubic inches which have the shape of right circular cylinders and be made of aluminum. Determine the radius and height of the container that contains the least amount of metal.

19. Use the definition of area, find the area of the region under the graph of $f(x) = 2x + 1$ on $[0, 2]$ by choosing C_k as the left end point.

20. Compute the Riemann sum for $f(x) = 2\sin x$, on $[0, \frac{5\pi}{4}]$, using the five subintervals, and choosing C_k as the right end point.

21. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$

22. Find the area of the surface obtained by revolving the graph of $y = \sqrt{x}$ on the interval $[4, 9]$ about the x-axis.

23. Find the center of mass of a system comprising three particles with masses 2, 3, and 5 slugs, located at the points $(-2, 2)$, $(4, 6)$ and $(2, -3)$ respectively. (Assume that all distances are measured in feet)

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (a) Write a short note on second derivative test.
(b) Find the relative extrema of $f(x) = x^3 - 3x^2 - 24x + 32$ using the second derivative test.

25. Sketch the graph of the function $f(x) = \frac{1}{1 + \sin x}$

26. State and prove both Part 1 and Part 2 of Fundamentl Theorem of Calculus.

27. Find the length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$

(2 × 10 = 20 Marks)

****