20U202A (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2021

(CBCSS - UG)

CC20U MTS2 C02 - MATHEMATICS - II

(Mathematics - Complementary Course)

(2020 Admission - Regular)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Graph the set of points whose polar coordinates satisfy the condition $0 \le r \le 1$.
2. State inverse function test.
3. Replace the Cartesian equation $y^2 = 4x$ by the equivalent polar equation.
4. Find $\frac{d}{dx}(x\sinh x - \cosh x)$.
5. Show that $\frac{d}{dx}\left(\cosh^{-1}x\right) = \frac{1}{\sqrt{x^2-1}}$.
6. Find $\lim_{n\to\infty}\sin\left(\frac{\pi n}{2n+1}\right)$.
7. Sum the series $\sum_{i=0}^{\infty}\frac{1}{5^i}$
8. Test for convergence of the series $\sum_{i=2}^{\infty}\frac{1}{\ln i}$
9. State the alternate series test.
10. Define the terms basis and dimension of a vectorspace.
11. If A is a triangular matrix of order 3, prove that det A is the product of its diagonal elements.
12. Determine whether the matrix $\begin{bmatrix} \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ is orthogonal.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Find the area of the surface generated by revolving the curve $y = x^3$, about the x-axis for $0 \le x \le \frac{1}{2}$.

14. Evaluate $\int_0^1 \ln x \, dx$.

15. Evaluate $\int_0^{\frac{\pi}{2}} (x + \sin x) \, dx$ using Riemann sums and trapezoidal rule with $n = 8$. Compare these two approximate values with the actual value.

16. Let $B = \{w_1, w_2, w_3\}$ where $w_1 = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$, $w_2 = \langle \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$, $w_3 = \langle 0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$. Show that B is an orthonormal basis. Express $u = \langle 3, -2, 9 \rangle$ in terms of B.

17. Find the rank of the matrix $\begin{bmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 0 & 1 & 0 & 0 \\ 2 & 5 & 6 & 8 \end{bmatrix}$

18. Find the eigen values and the correponding eigen vectors of the matrix $A = \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$.

19. Show that the matrix $\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ satisfies its own characteristic equation.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. Each question carries 10 marks.

20. Solve the linear system using Gaussian elimination or Gauss-Jordan method.

$$\begin{aligned} 2x_1 + 6x_2 + x_3 &= 7 \\ x_1 + 2x_2 - x_3 &= -1 \\ 5x_1 + 7x_2 - 4x_3 &= 9 \end{aligned}$$

21. Given $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, find a orthogonal matrix P that diagonalizes A and the diagonal matrix $D = P^T A P$

(1 × 10 = 10 Marks)

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