18U602 (Pages: 2) Name : ....

Reg. No : ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2021

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT6 B10/ CC18U MAT6 B10 - COMPLEX ANALYSIS

(Mathematics - Core Course)

(2015 Admission onwards)

Time : Three Hours Maximum : 120 marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. Compute $f'(z)$ where $f(z) = \exp(1 - i - 4i\, z^2)^3$.
2. Find P.V. of $(-i)^i$.
3. Obtain the real and imaginary parts of $\sin z$.
4. Show that any two antiderivatives of a complex function differ by a constant.
5. Obtain the value of $\int_{|z|=1} \exp(-z^2)\, dz$.
6. Define analyticity of a function at a point.
7. Determine whether the function $f(z) = 2z^2 - 3i - ze^{\cos z} + e^{-z}$ is entire. Justify
8. Compute the derivative of $\sinh z$.
9. State maximum modulus principle.
10. Write the Maclaurin series of $\frac{1}{1-z^2}$ when $|z| < 1$.
11. How do you differentiate a power series? What is the radius of convergence of the derived series?
12. Compute the residue of $\exp\left(\frac{1}{z}\right)$ at $z = 0$.

($12 \times 1 = 12$ Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Show that differentiable functions are continuous.
14. Determine at which points $f(z) = |z|^2$ is analytic.
15. Verify that $u(x, y) = e^{-y} \sin x$ is harmonic.
16. Use Cauchy Riemann equations in polar coordinates to compute the derivative of $f(z) = 1/z$.
17. Show that $e^{(z_1+z_2)} = e^{z_1} e^{z_2}$.
18. Obtain all the zeros of $\sin z$.
19. If $w(t)$ is a complex function of a real variable t, then show that $\operatorname{Re} \int_a^b w(t)\, dt = \int_a^b \operatorname{Re} w(t)\, dt$.

20. Evaluate the integral $\int_C f(z)\,dz$ where $f(z) = y - x - i3x^2$ and C is the line segment joining 0 and $1+i$.

21. Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$. Show that $|\int_C \frac{z+4}{z^3-1}\,dz| \leq 6\pi/7$.

22. Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate $\int_C \frac{\cosh z}{z^4}\,dz$.

23. State and prove Morera's theorem.

24. Find the Laurent's series that represents the function $f(z) = z^2 \sin(\frac{1}{z^2})$ in the domain $0 < |z| < \infty$.

25. What is an essential singularity? Give an example.

26. Determine the order of the pole of $f(z) = \frac{\tanh z}{z^2}$.

($10 \times 4 = 40$ Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. Suppose $f'(z) = 0$ in a (connected) domain. Show that $f(z)$ is constant.

28. Obtain the harmonic conjugate of $u(x, y) = y^3 - 3x^2y$. Obtain an analytic function with $u(x, y)$ real part.

29. Define logarihmic function. Obtain a domain in which it is analytic and find its derivative.

30. True or false? Mean value theorem for derivatives hold for complex functions. Justify.

31. Evaluate $\displaystyle\int_{|z|=1} \frac{1}{z}\,dz$.

32. State and prove principle of deformation of paths.

33. State and prove Liouville's theorem.

34. Evaluate $\int_C \frac{dz}{z(z-2)^4}$ where C is the positively oriented circle $|z - 2| = 1$.

35. Evaluate using the method of residues: $\displaystyle\int_0^{2\pi} \frac{1}{3 + 2\cos\theta}\,d\theta$

($6 \times 7 = 42$ Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. Derive Cauchy Riemann equations with necessary assumptions.

37. Show that a continuous function $f(z)$ defined on a domain D has an antiderivative if and only if the value the integral $\int_C f(z)\,dz = 0$ for every closed contour C lying in D .

38. Obtain the Laurent series expansion of $f(z) = \frac{-1}{(z-1)(z-2)}$ valid in the domains $|z| < 1$, $1 < |z| < 2$ and $2 < |z| < \infty$.

($2 \times 13 = 26$ Marks)

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