18U603 (Pages: 3) Name: ……………………….

Reg. No………….…….………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2021

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT6 B11/ CC18U MAT6 B11 - NUMERICAL METHODS

(Mathematics - Core Course)

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. By the method of false position, write the first approximation to the root of $f(x) = 0$.
2. Write the secant formula for finding a root of $f(x) = 0$.
3. Define the mean operator and write a relation connecting μ and E.
4. Show that $\Delta = E\nabla$.
5. State Gauss's backward interpolation formula.
6. Define divided differences for the points $(x_0, y_0), \ldots\ldots (x_n, y_n)$.
7. Write the formula for computing $\left.\frac{dy}{dx}\right]_{x_0}$, given a set of n values of (x, y).
8. State the general formula for numerical integration.
9. Find the characteristic equation of the matrix $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$.
10. Define spectral radius of a matrix.
11. Write Milne's predictor formula.
12. Write the fourth order Runge-Kutta formula for solving a first order initial value problem.

(12 × 1 = 12 Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Explain the method of iteration to find a root of $f(x) = 0$.
14. Solve $x^3 - 6x + 4 = 0$ to find a root between 0 and 1 using Newton Raphson method.
15. Show that (i) $\Delta = \nabla E = \delta E^{1/2}$ (ii) $E = e^{hD}$, where E is the shift operator and D is the differential operator.
16. Prove that the n^{th} divided differences of a polynomial of degree n is a constant.
17. Construct Newton's forward interpolation polynomial for the data:

x	4	6	8	10
$f(x)$	1	3	8	16

Turn Over

18. Evaluate $\sqrt{153}$ by Lagrange's interpolation formula from the table below:

x	150	152	154
y	12.247	12.329	12.410

19. Use Gauss' forward formula to find $f(32)$ from the following data:

x	25	30	35	40
$f(x)$	0.2707	0.3027	0.3386	0.3794

20. Explain Trapezoidal rule of integration.

21. Decompose the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ in the LU form.

22. Solve the following system by Gauss elimination method.

$$x + y + z = 7$$
$$x + 2y + 3z = 16$$
$$x + 3y + 4z = 22$$

23. Solve the IVP $\frac{dy}{dx} = \frac{1}{x^2+y}$, $y(4) = 4$ using Taylor's series method. Find $y(4.1)$.

24. For $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, find $y(0.1)$ by Runge-Kutta second order formula.

25. For the differential equation $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Compute $y(1.4)$ by Adams-Bashforth method.

26. Write down the difference between Jacobi's method and Gauss-Seidel method.

(10 × 4 = 40 Marks)

Section C

Answer any ***six*** questions. Each question carries **7** marks.

27. Using Ramanujan's method, find a root of $\sin x = 1 - x$.

28. Find a positive root of $xe^x = 1$ between 0 and 1 with tolerance 0.05% by bisection method.

29. Find x for $\sinh x = 62$ from the following table:

x	4.80	4.81	4.82	4.83	4.84
$y = \sinh x$	60.7511	61.3617	61.9785	62.6015	63.2307

30. A rod is rotating in a plane about one end. The table gives the angle θ in radians at t seconds. Find the angular velocity at $t = 0.7$ seconds.

t	0.0	0.2	0.4	0.6	0.8	1.0
θ	0.0	0.12	0.48	1.10	2.0	3.20

31. Solve the following system by LU decomposition.

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

32. Find the inverse of the coefficient matrix using Gauss method:

$$3x + 2y + 4z = 7$$
$$2x + y + z = 4$$
$$x + 3y + 5z = 2$$

33. Determine the largest eigen value and the corresponding eigen vector of the matrix

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

34. Find $y(0.5)$ by modified Euler's method with $h = 0.1$ to solve the IVP

$$\frac{dy}{dx} = x + y^2, \; y(0) = 1.$$

35. Using Picard's method solve $\frac{dy}{dx} = x(1 + x^3 y)$, $y(0) = 3$ and find $y(0.2)$.

(6 × 7 = 42 Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. Find the number of students who obtained marks between 60 and 70 using Gauss' backward formula from the table below:

Marks	0-40	40-60	60-80	80-100	100-120
No: of students	250	120	100	70	50

37. Solve the system of equations by Gauss Jordan method.

$$10x - 2y - z - w = 3$$
$$-2x + 10y - z - w = 15$$
$$-x - y + 10z - 2w = 27$$
$$-x - y - 2z + 10w = -9$$

38. Evaluate $\int_0^6 \frac{1}{1+x^2}\, dx$ with $h = 1$ using

(a) Trapezoidal rule (b) Simpson's $1/3$ rule (c) Simpson's $3/8$ rule.

(2 × 13 = 26 Marks)

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