**21P105** (Pages: 2) Name: ……………….………

Reg. No: ……………..…...

**FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021**

(CBCSS-PG)

(Regular/Supplementary/Improvement)

**CC19 MTH1 C05 – NUMBER THEORY**

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

## PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that if f and g are multiplicative, so is their Dirichlet product f*g
2. Find all integers n such that $\phi(n)=12$
3. If f is multiplicative then prove that f(1)=1
4. Write a brief sketch of an elementry proof of the prime number theorem.
5. State Shapiro's Tauberian theorem.
6. Evaluate the Legendre's symbol (3/383).
7. Distinguish between plain text and cipher text.
8. Prove that 5 is a quadratic residue of an odd prime p if $p \equiv \pm 1 (\text{mod} 10)$

**(8 × 1 = 8 Weightage)**

## PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT 1

9. State and prove Legendre's identity.
10. State and prove Euler's summation formula.
11. Prove that $\frac{n}{\phi(n)} = \sum_{d/n} \frac{\mu^2(d)}{\phi(d)}$

UNIT II

12. Prove that the relation $M(x) = O(x)$ as $x \longrightarrow \infty$ implies $\psi(x) \sim$ x as $x \longrightarrow \infty$ .
13. Prove that $\lim_{x \to \infty} \left(\frac{M(x)}{x} - \frac{H(x)}{x \log x}\right) = 0$.
14. Prove that there is a constant A such that

$$\sum_{p \le x} \frac{1}{p} = \log(\log x) + A + O\left(\frac{1}{\log x}\right), \quad \forall\, x \ge 2\,.$$

UNIT III

15. If p and q are distinct odd primes then prove that that (p/q)(q/p)=(−1)(p−1)(q−1)4
16. In the 27 letter alphabet (with blank = 26), use the affine enciphering transformation with key a = 13, b = 9 to enchipher the message "HELP ME"
17. How will you authenticate a message in public key cryptosystem?

**(6 × 2 = 12 Weightage)**

## PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Prove that if both g and f∗g are multiplicative then f is also multiplicative and hence show that the set of all multiplicative function is a subgroup of the group of all arithmetical functions f with f(1) ≠ 0
19. Let {a(n)}be a non negative sequence such that $\sum_{n<x} a(n)\left[\frac{x}{n}\right] = x\ \log x + O(x),\ \forall\ x\ \geq 1.$ Prove that there is a constant B > 0 such that: $\sum n \leq x\ a(n) \leq nB(x)$ for all $x \geq 1$.
20. State and prove Abel's identity.
21. State and prove Quadratic Reciprocity low.

**(2 × 5 = 10 Weightage)**

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