

21U202S

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Name:

Reg. No:

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CUCBCSS-UG)

CC15U MAT2 C02 – MATHEMATICS

(Mathematics - Complementary)

(2015 to 2018 Admissions -Supplementary/Improvement)

Time: Three Hours

Maximum: 80 Marks

PART - A

Answer *all* questions. Each question carries 1 mark.

1. Prove that $\cosh \frac{x}{2} = \sqrt{\frac{\cosh x + 1}{2}}$
2. $\int \tanh 5x \, dx = \dots$
3. A function is said to be smooth if
4. Discuss the convergence of $\int_1^{\infty} \frac{1}{x^2} \, dx$
5. Find the n^{th} term of the sequence 2,1,4,3,6,5,8,7,
6. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{n}$
7. Define the radius of convergence of a power series.
8. Replace the Cartesian equation $y^2 = 4x$ by equivalent polar equation.
9. Identify the conic $r = \frac{12}{1-5 \sin \theta}$
10. Convert the rectangular coordinate $(-1,1,2)$ into cylindrical coordinates.
11. Find the boundary of the domain of the function $f(x, y) = \sqrt{y - x^2}$
12. $\lim_{(x,y) \rightarrow (1,1)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2} = \dots$

(12 × 1 = 12 Marks)

PART - B

Answer any *nine* questions. Each question carries 2 marks.

13. Prove that $\cosh^2 x - \sinh^2 x = 1$
14. Differentiate $\cosh^{-1} \sqrt{x+1}$ w.r.t x .
15. Evaluate $\lim_{n \rightarrow \infty} \frac{n-1}{n+1}$
16. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n}$
17. Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$ Converges absolutely
18. Find the Taylor series generated by $f(x) = e^x$ at $x = 1$.

19. If $f(x) = \frac{1}{1-x}$, find series for $f'(x)$ and $f''(x)$.
20. Find the points of intersection of the curves $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$
21. Sketch the surface $r = 1 + \cos \theta$
22. Find $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.
23. Find $\frac{dy}{dx}$ if $xe^y + \sin xy + y - \ln 2 = 0$
24. Find the linearization of $f(x, y) = x^2 + y^2 + 1$ at the point $(1,1)$.
- (9 × 2 = 18 Marks)**

PART - C

Answer any *six* questions. Each question carries 5 marks.

25. Show that $\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$ for all $-1 < x < 1$.
26. Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the lines $y = 0, x = 2$ about the x -axis.
27. Find the length of the astroid $x^{2/3} + y^{2/3} = 1$
28. Prove that $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ is convergent.
29. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$
30. For what values of x does the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converges?
31. Find the area of the region enclosed by the cardioid $r = 1 - \cos \theta$
32. Show that the function $f(x, y) = \frac{x}{\sqrt{x^2+y^2}}$ has no limit as $(x, y) \rightarrow (0,0)$.
33. Find the linearization of $f(x, y, z) = xz - 3yz + 2$ at the point $(1,1,2)$. Find an upper bound for the error incurred in replacing $f(x, y, z)$ by the linearization on the region $|x - 1| \leq 0.01, |y - 1| \leq 0.01, |z - 2| \leq 0.02$

(6 × 5 = 30 Marks)

PART - D

Answer any *two* questions. Each question carries 10 marks.

34. Find the area of the surface generated by revolving the curve $y = x^3, 0 \leq x \leq 1/2$, about the x -axis.
35. Discuss the converges of $\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots$ for $p > 0$.
36. Find the length of the cardioid $r = a(1 + \cos \theta)$.

(2 × 10 = 20 Marks)
