21U272S (Pages: 2) Name. . . . .

Reg.No. . . . .

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CUCBCSS-UG)

CC15U ST2 C02 - PROBABILITY DISTRIBUTIONS

(Statistics - Complementary Course)

(2016 to 2018 Admissions - Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

Section A

[One word questions. Answer ***all*** questions. Each question carries 1 mark]

Fill up the blanks:

1. If C is a constant $V(C) = \cdots\cdots\cdots$
2. As $x \to -\infty$, the joint cumulative distribution $F(x, y)$ of a bivariate random variable (X, Y) becomes $\cdots\cdots\cdots$
3. For any finite real numbers a and b, $E(X + a) = b$. Then $E(X) = \cdots\cdots\cdots$
4. The discrete distribution possessing the memoryless property is $\cdots\cdots\cdots$
5. A family of distribution for which the mean is equal to variance is $\cdots\cdots\cdots$

Write true or false:

6. If X and Y are independent random variables, the $f(x, y) = f_1(x)f_2(y)$.
7. A curve is leptokurtic when $\beta_2 > 0$.
8. m.g.f does not exist always.
9. $E(X^2) \geq (E(X))^2$.
10. In a normal distribution M.D $= \frac{4}{5}$ S.D.

(10 × 1 = 10 Marks)

Section B

[One Sentence questions. Answer ***all*** questions. Each question carries 2 marks]

11. Define conditional variance of a random variable X given Y.
12. Define Mathematical expectation.
13. Define characteristic function.
14. Define m.g.f and how do you determine moments from it?
15. Define Kurtosis.
16. Find the m.g.f. of a Binomial distribution.
17. Define Cauchy distribution.

(7 × 2 = 14 Marks)

Section C

[Paragraph questions. Answer any ***three*** questions. Each question carries 4 marks]

18. Define bivariate probability distribution (X, Y) where X and Y are discrete random variables.

19. If X and Y are two random variables with joint $p.d.f$ $f(x,y) = \dfrac{x+2y}{18}$ where $(x,y) = (1,1),\ (1,2),\ (2,1),\ (2,2)$ and 0 otherwise. Are the variables independent?

20. Define Gamma distribution. Establish its additive property.

21. In two independent random variables X and Y show that $M_{X+Y}(t) = M_X(t)M_Y(t)$.

22. Derive the mean and variance of Poisson distribution.

(3 × 4 = 12 Marks)

Section D

[Short Essay questions. Answer any ***four*** questions. Each question carries 6 marks]

23. If $f(x,y) = 2(x + y - 3x^2y)$, $0 < x < 1$ and $0 < y < 1$ find the marginal and conditional pdfs.

24. Prove that for any two r.v.s X and Y, $[E\,(XY)]^2 \leq E\,(X^2)\,E\,(Y^2)$.

25. State and prove the addition theorem on expectation.

26. Establish Renovsky formula.

27. Establish the lack of memory property of exponential distribution.

28. Obtain the mean of normal distribution.

(4 × 6 = 24 Marks)

Section E

[Essay questions. Answer any ***two*** questions. Each question carries 10 marks]

29. Derive the relation between raw and central moments and hence express first four central moments in terms of raw moments.

30. Derive the recurrence relation for a Poisson distribution for central moments and hence obtain skewness and kurtosis.

31. If the joint pdf of X and Y, is $f(x,y) = x + y,\ \ 0 \leq x \leq 1,\ \ 0 \leq y \leq 1$. Find the coefficients of correlation and regression.

32. **(i)** State and prove Chebyshev's inequality. **(ii)** State Bernoulli's law of large numbers.

(2 × 10 = 20 Marks)

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