

**FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022**

(CUCBCSS-UG)

**CC15U MAT4 C04/ CC18U MAT4 C04 - MATHEMATICS IV**

(Mathematics - Complementary Course)

(2015 to 2018 Admissions - Supplementary/Improvement)

Time: 3 Hours

Maximum: 8 Marks

**Section A**Answer *all* questions. Each question carries 1 mark.

1. Solve the differential equation  $y'' - y = 0$
2. Find the Wronskian of  $e^{ax}$  and  $e^{bx}$
3. Find a particular solution of  $y'' - y' - 2y = 6e^x$
4. Find  $\mathcal{L}(t^2)$
5. Evaluate  $\mathcal{L}^{-1}\left(\frac{4}{(s-2)^2}\right)$
6. Find the Laplace transform of the Dirac delta function.
7. What is the fundamental period of  $\cos t$ ?
8. Give an example of a function which is neither even nor odd.
9. At a point of discontinuity  $x_0$ , the Fourier series of a periodic function converges to .....
10. Write down the error estimate of Simpson's rule.
11. Write down the one dimensional heat equation.
12. Show that  $u = 2xy$  is a solution of the Laplace equation.

**(12 × 1 = 12 Marks)****Section B**Answer any *nine* questions. Each question carries 2 marks.

13. Find a general solution of  $y'' - 4y' + 4y = 0$
14. Solve  $x^2y'' - 4xy' + 6y = 0$
15. Show that  $y = e^x[2\sin x - 4\cos x]$  is a particular solution of  $(D^2 + 1)y = 10e^x \sin x$
16. Find  $\mathcal{L}(\sin 2t \sin 3t)$
17. Prove that  $f * g = g * f$ .
18. Find  $\mathcal{L}^{-1}\left(\frac{e^{-3s}}{(s-1)^4}\right)$
19. Find  $a_0$  in the Fourier series expansion of  $f(x) = x^2$ ,  $-\pi < x < \pi$ .
20. Find the half range sine series of  $f(x) = \pi - x$ ,  $0 < x < \pi$ .

21. Is  $u = e^{2t} \cos x$  a solution of the wave equation?
22. Prove that the solution of the partial differential equation  $u_{xx} + 4u = 0$  can be taken as  $u = v(y) \cos 2x + w(y) \sin 2x$
23. Using Picard's method find an approximate solution of  $y' = x + y, y(0) = 1$ .
24. Use the Trapezoidal rule with  $n = 4$  to estimate  $\int_0^2 \frac{1}{1+x} dx$

**(9 × 2 = 18 Marks)**

### Section C

Answer any **six** questions. Each question carries 5 marks.

25. Find a general solution of  $y'' - 3y' + 2y = 4x + e^{3x}$
26. Using the method of variation of parameters solve  $y'' + y = \csc x$ .
27. Find the Laplace transform of  $f(t) = \frac{1 - e^t}{t}$
28. Find the inverse Laplace transform of  $\ln \frac{s+a}{s+b}$
29. Find a Fourier series to represent  $f(x)$  in the interval  $(-\pi, \pi)$  where  $f(x) = \begin{cases} -k, & \text{if } -\pi < x < 0 \\ k, & \text{if } 0 \leq x < \pi \end{cases}$
30. Find the Fourier series for  $|x|$  in  $[-\pi, \pi]$ . Deduce that  $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
31. By separating the variables solve  $u_x = 2u_y + u$
32. Find the deflection  $u(x, t)$  of the string of length  $L = \pi$  when  $c^2 = 1$ , the initial velocity is zero and the initial deflection is  $k[\sin x - (1/2) \sin 2x]$ .
33. Using the improved Euler's method solve the initial value problem  $y' = x + y, y(0) = 0$  by choosing  $h = 0.2$  and computing  $y_1, y_2, \dots, y_5$ .

**(6 × 5 = 30 Marks)**

### Section D

Answer any **two** questions. Each question carries 10 marks.

34. Solve the initial value problem  $y'' + 2y' + 5y = 1.25e^{0.5x} + 40 \cos 4x - 55 \sin 4x, y(0) = 0.2$  and  $y'(0) = 60.1$
35. Using Laplace transforms solve  $y'' + 2y' + 5y = e^{-t} \sin t, y(0) = 0, y'(0) = 1$
36. Find the Fourier series of the function  $f(x) = \pi \sin \pi x, 0 < x < 1$  with period  $2p = 1$ .
37. Apply the Runge-Kutta method with  $h = 0.1$  to find  $y(0.2)$  for the initial value problem  $y' = x + y^2, y(0) = 1$ .

**(2 × 10 = 20 Marks)**

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