20U401 (Pages: 2) Name: ....

Reg.No: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS4 B04 / CC20U MTS4 B04 - LINEAR ALGEBRA

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** qestions. Each question carries 2 marks.

1. Is the system of equations $x - y = 1;\ 2x + y = 6$ is consistent?
2. Define trivial solution of $AX = 0$
3. Solve by inverting the coefficient matrix $4x - 3y = -3; 2x - 5y = 9$
4. Find A^{-1}, if $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$
5. If A and B are two square matrices, is it true that $det(A + B) = det(A) + det(B)$? Justify your answer.
6. Show that the lines through origin are subspaces of $\mathbb{R}^2$
7. Prove that a finite set that contains 0 is linearly dependent.
8. State Plus-Minus Theorem.
9. Define column vectors.
10. Use matrix multiplication to find the reflection of $x = (-1, 2)$ about the line $y = x$
11. Define 'kernel' of a matrix transformation.
12. Find the equation of the image of the line $y = -4x + 3$ under the multiplication by matrix $A = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$
13. Define Eigen Value.
14. Define unit vector in an inner product space.
15. Check whether $u = (1, 1)$ and $v = (1, -1)$ are orthogonal with respect to the Euclidean inner product in $\mathbb{R}^2$

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** qestions. Each question carries 5 marks.

16. Show that if a square matrix A satisfying the equation $A^2 + 2A + I = 0$, then A must be invertible.What is the inverse.
17. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.For what value of a, b, c and d, the matrices A and B are commute.
18. Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation.Find the standard matrix for the transformation and find $T(x), if\ T(e_1) = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}$, $T(e_3) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $X = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$
19. Show that the set $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ forms a basis for $\mathbb{R}^3$
20. Consider the bases $B = \{u_1, u_2\}$ and $B' = \{u'_1, u'_2\}$ where $u_1 = (2, 2), u_2 = (4, -1), u'_1 = (1, 3), u'_2 = (-1, 1)$ find the transition matrix from B' to B
21. Show that $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$ is not diagonalizable.
22. Let the vector space P_2 have inner product $\langle f, g \rangle = \int_a^b f(x).\,g(x)$ Apply Gram-Schmidt process to transform the basis $\{1, x, x^2\}$ into an orthogonal basis.
23. Find the spectral decompostion of $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (a) Prove that $\begin{vmatrix} a_1 + b_1 t & a_2 + b_2 t & a_3 + b_3 t \\ a_1 t + b_1 & a_2 t + b_2 & a_3 t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

    (b) If B is the matrix that results when a single row or single column of A is multiplied by a scalar k then prove that $det(B) = k.\,det(A)$
25. Show that the spaces $R^2\ and\ R^3$ are vector spaces.
26. (a) State and prove Dimension Theorem.

    (b) Verify Dimension Theorem for $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -3 & 3 \\ 4 & 8 & -4 & 4 \end{bmatrix}$
27. Prove that the following are equivalent for an $n \times n$matrix A.

    (a) A is orthogonal.

    (b) The row vectors of A form an orthonormal set in $\mathbb{R}^n$with the Euclidean inner product.

    (c) The Column vectors of A form an orthonormal set in $\mathbb{R}^n$ with the Euclidean inner product.

(2 × 10 = 20 Marks)

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