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Name: .....

Reg. No: .....

**SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022**

(CBCSS-UG)

**CC19U MTS6 B14 - GRAPH THEORY**

(Mathematics - Elective Course)

(2019 Admission - Regular)

Time: 2 Hours

Maximum: 60 Marks

Credit: 2

**Section A**

Answer *all* questions. Each question carries 2 marks.

1. Define self-complementary graph. Give an example.
2. Define incidence matrix of a graph  $G$ .
3. Define the eccentricity of a vertex  $v$  in a graph  $G$ . What is the radius of a graph  $G$ ?
4. Define vertex connectivity of a graph. What is the connectivity of  $K_n$ ?
5. How many vertices and edges are there for the  $k$ -cube graph  $Q_k$ ?
6. Show that it is impossible to have a group of 9 people at a party such that each one knows exactly 5 of the others in the group.
7. State Whitney's theorem.
8. Is the complete graph  $K_5$  Euler? Justify your answer.
9. Define Hamiltonian graph. Give an example for non-Hamiltonian graph.
10. State Euler's formula for planar graphs and verify it for  $K_4$ .
11. Define a bridge. How many bridges are there in a path having  $n$  vertices?
12. Write the wheel graph as a join of two graphs.

**(Ceiling: 20 Marks)**

**Section B**

Answer *all* questions. Each question carries 5 marks.

13. Draw all trees with 5 vertices.
14. Prove that for any simple graph  $G$ , there is an even number of odd degree vertices.
15. Let  $G$  be a graph with  $n$  vertices where  $n \geq 2$ . Then prove that  $G$  has at least two vertices which are not cut vertices.
16. Let  $G$  be a graph in which the degree of every vertex is at least two. Then prove that  $G$  contains a cycle.
17. Prove that an edge  $e$  of a graph  $G$  is a bridge if and only if  $e$  is not a part of any cycle in  $G$ .

18. If  $T$  is a tree with  $n$  vertices then prove that it has precisely  $n - 1$  edges.

19. Prove that a graph is connected if and only if it has a spanning tree.

**(Ceiling: 30 Marks)**

### **Section C**

Answer any *one* question. Each question carries 10 marks.

20. Prove that a non empty graph with atleast two vertices is bipartite if only if it has no odd cycles.

21. a) Prove that  $K_5$  is non-planar.

b) Prove that  $K_{3,3}$  is non-planar.

**(1 × 10 = 10 Marks)**

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