19U602S (Pages: 2) Name: …..…………..……….

Reg. No: …………..……….

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CUCBCSS-UG)

CC15U MAT6 B09 / CC18U MAT6 B09 - REAL ANALYSIS

(Mathematics – Core Course)

(2016 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 120 Marks

Part-A

Answer ***all*** questions. Each question carries 1 mark**.**

1. State boundedness theorem.
2. State true or false: Continuous functions are always bounded.
3. Does the function $g(x) = x^3 - 3x$ has a root in [1,2]?
4. Give an example of a continuous function which is not uniformly continuous.
5. Calculate the norm of the partition $\wp = (0,1,2,4)$.
6. Show that every constant function is Riemann integrable.
7. Find lim $\left(\frac{x^2+nx}{n}\right)$.
8. Define uniform convergence of a sequence of functions $\{f_n(x)\}$.
9. Find the radius of convergence of $\sum x^n$.
10. Give an example of a second kind improper integral.
11. Define Beta function.
12. What is the value of $\Gamma(\frac{1}{2})$?

(12 × 1 = 12 Marks)

Part-B

Answer any ***ten*** questions. Each question carries 4 marks**.**

13. Define absolute maximum point and absolute minimum point for $f: A \to \mathbb{R}$; $A \subseteq \mathbb{R}$
14. Define Lipschitz function. Prove that a Lipschitz function $f: A \to \mathbb{R}$ is uniformly continuous on A.
15. Show that $g(x) = \frac{1}{x}$ is uniformly continuous on $A = \{x \in R: 1 \leq x \leq 2\}$.
16. Define Riemann sum and Riemann integral of a function f on an interval $[a, b]$ corresponding to a tagged partition $\dot{\mathcal{P}}$.
17. If $f, g \in R[a, b]$, show that $f + g \in R[a, b]$, and $\int_a^b f + g = \int_a^b f + \int_a^b g$.
18. State substitution theorem for Riemann integration. Use it to evaluate $\int_1^4 \frac{cos\sqrt{t}}{\sqrt{t}} dt$.
19. Use the fundamental theorem of calculus to evaluate $\int_2^5 (2x^2 + 3x + 1)dx$.
20. Discuss the pointwise convergence of the sequence of functions (x^n).

21. State and prove Weierstrass M- Test for a series of functions.

22. Test the convergence of $\int_1^{\infty} \frac{lnx}{x^2} dx$.

23. Find $\int_0^2 (8 - x^3)^{\frac{-1}{3}} dx$

24. Express $\int_0^1 \frac{x^2}{\sqrt{1-x^2}} dx$ in terms of Beta function.

25. Show that $\beta(m,n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$.

26. Evaluate $\int_0^{\frac{\pi}{2}} Sin^7\theta Cos^5\theta \, d\theta$.

(10 × 4 = 40 Marks)

Part-C

Answer any ***six*** questions. Each question carries 7 marks**.**

27. If f and g are uniformly continuous on a subset A of $\mathbb{R}$, then prove that $f + g$ is uniformly continuous on A.

28. Let I be an interval and let $f: I \to \mathbb{R}$ be continuous on I. Then prove that $f(I)$ is an interval.

29. Let $f(x) = x, x\epsilon[0,1]$. Show that $f \epsilon R[a,b]$.

30. If $f: [a,b] \to \mathbb{R}$ is monotone on $[a,b]$, then show that $f \epsilon R[a,b]$.

31. State and prove Squeeze theorem of Riemann integrable functions.

32. State and prove Cauchy criterion for the uniform convergence of the sequence of functions.

33. Define Cauchy principal value of $\int_{-\infty}^{\infty} f(x)dx$. Also evaluate it for $\int_{-\infty}^{\infty} Sinx \, dx$.

34. Show that $\int_a^{\infty} \frac{1}{x^p} \, dx$, $a > 0$ converges if $p > 1$ and diverges if $p \leq 1$.

35. Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(6 × 7 = 42 Marks)

Part-D

Answer any ***two*** questions. Each question carries 13 marks**.**

36. (a) State and prove intermediate value theorem.

    (b) Prove that $\sin x$ is uniformly continuous on $[0, \infty)$.

37. (a) Define indefinite integral of f where $f \epsilon R[a,b]$ with a base point a.

    (b) State and prove fundamental theorem of calculus (Second form).

38. (a) Find the value of $\int_2^3 (x-2)^2(3-x)^3 dx$.

    (b) Check the integrability of Dirichlet function.

(2 × 13 = 26 Marks)

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