19U604S (Pages: 2) Name: ....

Reg. No.: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CUCBCSS-UG)

CC15U MAT6 B10/ CC18U MAT6 B10 - COMPLEX ANALYSIS

(Mathematics - Core Course)

(2015 to 2018 Admissions - Supplementary/Improvement)

Time: 3 Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. What is the imaginary part of $f(z) = z^2$?
2. What do you meant by an entire function?
3. Express Cauchy-Riemann equations in polar form.
4. The value of $e^{i\pi} =$ .....
5. $\sin(iy) =$ .....
6. $\text{Log}(-ei) =$ .....
7. Parametrize the unit circle $|z| = 1$.
8. Give an example for a simply connected domain.
9. The region of convergence of the power series $1 + z + z^2 + ...$ is ......
10. Evaluate $\oint_{|z|=1} \frac{z^2+4}{z-2} dz$
11. For the function $f(z) = \frac{1-\cosh z}{z^4}$, $z = 0$ is a pole of order .....
12. If $f(z) = \frac{2}{(z-1)(z+4)}$, find residue of $f(z)$ at $z = 1$.

($12 \times 1 = 12$ Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Prove that if $f(z)$ is differentiable at z_0 then it is continuous at z_0.
14. Show that $f(z) = \bar{z}$ is nowhere analytic.
15. Suppose the function $f(z) = u + iv$ is analytic in a domain D. Prove that u and v are harmonic in D.
16. Find all values of z so that $e^z = -2$.
17. Prove that $|sinz|^2 = \sin^2 x + \sinh^2 y$.
18. Find the principal value of $(-i)^i$
19. Evaluate $\int_C \bar{z} dz$ where C is the right hand half of the circle $|z| = 2$ from $z = -2i$ to $z = 2i$.

20. Evaluate $\int_C \frac{1}{z}dz$ where C is the circle $|z| = \frac{1}{2}$.

21. State and prove *Cauchy's inequality*.

22. Prove that $\sum_{n=0}^{\infty} \frac{(3-4i)^n}{n!}$ is absolutely convergent.

23. Obtain a Taylor series representation of $f(z) = \frac{1}{z}$ about $z = 1$.

24. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!z^n}{n^n}$

25. What are the singular points of the function $f(z) = \frac{1}{\sin(\pi/z)}$? Give its classification.

26. Find the residue at all the singular points of the function $f(z) = \cot z$.

(10 × 4 = 40 Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. Derive *Cauchy-Riemann equations* with necessary assumptions.

28. Show that the function $u(x, y) = 4xy - x^3 + 3xy^2$ is harmonic. Find the harmonic conjugate and the most general analytic function $f(z)$ with u as real component.

29. By taking $z_1 = z_2 = -1$, verify $\log z_1 z_2 = \log z_1 + \log z_2$. Is the property valid if we replace $\log z$ by the principal branch Log z?

30. State and prove *ML-inequality*. Without evaluating the integral show that $\left|\int_C \frac{dz}{z^2-1}\right| \leq \frac{\pi}{3}$ where C be arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ lying in the first quadrant.

31. Evaluate $\int_C \frac{1}{z \sin z}dz$ where C is the unit circle $|z| = 1$ oriented in the positive direction.

32. State and prove *fundamental theorem of algebra*.

33. Show that the power series $\sum_{n=1}^{\infty} na_n z^{n-1}$ obtained by differentiating the power series $\sum_{n=0}^{\infty} a_n z^n$ term by term has the same radius of convergence as the original series.

34. *State and prove Cauchy's residue theorem.* Using residue theorem, evaluate $\int_C \frac{1}{z^3(z+4)}dz$ where C is the circle $|z| = 2$.

35. Using residue theorem evaluate $\int_0^{2\pi} \frac{1}{13 - 5\sin\theta}d\theta$

(6 × 7 = 42 Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. State and prove *Cauchy's integral formula.*

37. Find all Laurent series representations of the function $f(z) = \frac{1}{1-z^2}$ with center at $z = 1$.

38. Evaluate the improper integral $\int_0^{\infty} \frac{1}{x^4+1}dx$

(2 × 13 = 26 Marks)

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