19U608S (Pages: 2) Name: ....

Reg. No.: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CUCBCSS-UG)

CC18U MAT6 B12 - NUMBER THEORY AND LINEAR ALGEBRA

(Mathematics - Core Course)

(2018 Admission - Supplementary/Improvement)

Time: 3 Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. Express 270 in canoincal form
2. Find gcd of the numbers 270 and 348.
3. Define Euler's phi function.
4. Write the remainder when 22! is divided by 23.
5. Write the dimension of the space of 2×2 matrices over the field $\mathbb{R}$.
6. Write a basis of the polynomials of degree ≤ 4 over the field $\mathbb{R}$.
7. Write a proper, non-trival subspace of $\mathbb{R}^2$ over $\mathbb{R}$.
8. Find the number of divisors of 720.
9. What is Euclid's lemma?
10. Write a map $f : \mathbb{R} \to \mathbb{R}$ which is not linear.
11. If $f : \mathbb{R}^2 \to \mathbb{R}$ is given by $f(x, y) = x + y$, for $x, y \in \mathbb{R}$, then find ker$f$.
12. Give a vector space which is isomorphic to $\mathbb{R}^4$ over $\mathbb{R}$.

($12 \times 1 = 12$ Marks)

Section B

Answer any ***ten*** question. Each question carries 4 marks.

13. Find the gcd of the numbers 36 and 84 and express it as a linear combination of 36 and 84.
14. Prove or disprove :There exist infinitely many primes.
15. Find img$f$, where f is the derivative map from $\mathbb{R}_2[X]$ to $\mathbb{R}_2[X]$.
16. Let V and W be two vector spaces over the field F and let f be an injective linear map from V to W. Prove or disprove: If $\{x_1, x_2, ..., x_n\}$ is a linearly indepedent subset of V, then $\{f(x_1), f(x_2), ..., f(x_n)\}$ is a linearly independent subset of W.
17. Check whether $(2, 3, 4)$ and $(1, 2, 3)$ are linearly independent in $\mathbb{R}^3$ over $\mathbb{R}$.
18. Find the number and sum of divisors of 820.
19. Show that the map $f : \mathbb{R}^2 \to \mathbb{R}_2[X]$ given by $f(a, b) = a^2 + bx$ is not a linear.
20. If p is a prime and $p|_{ab}$, then prove that $p|_a$ or $p|_b$.

21. If V has a finite basis, then prove that all linearly independent subsets of V are finite.

22. Prove that the intersection of any two subspaces of a vector space is again a vector space.

23. If $f : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $f(a, b) = (b, 0)$, then prove that ker$f$=img$f$

24. If p is prime number and $k > 0$, then prove that $\phi(p^k) = p^k\left(1 - \frac{1}{p}\right)$.

25. Is the converse of Fermat's little theorem true? Justify.

26. Define dimension of a vector space. Give example of a vector space of infinite dimension.

($10 \times 4 = 40$ Marks)

Section C

Answer any ***six*** question. Each question carries 7 marks.

27. Use congruency theory to establish that 7 divides $5^{2n} - 3 \times 2^{5n-2}$ for any natural number n.

28. Solve the congruence

$$x \equiv 2(mod3)$$
$$x \equiv 3(mod5)$$
$$x \equiv 2(mod7)$$

29. State and prove Wilson's Theorem.

30. Prove or disprove : Set of all 2×2 upper triangular matrices is a subspace of the set of all 2×2 matrices over $\mathbb{R}$

31. Using congruences, solve the Diophantine equation, $56x + 72y = 40$

32. State and prove Fermat's little theorem.

33. Show that a linear map $f : U \to V$ is injective if and only if ker$f = \{0_v\}$.

34. Solve the linear congruence $36x \equiv 8(mod102)$.

($6 \times 7 = 42$ Marks)

Section D

Answer any ***two*** question. Each question carries 13 marks.

35. State and prove Dimension Theorem

36. State and prove the Fundamental Theorem of Arithmetic.

37. Solve the linear congruences

$$17x \equiv 9(mod276)$$

($2 \times 13 = 26$ Marks)

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