19U601 (Pages: 2) Name: ……………………

Reg. No……………………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS-UG)

CC19U MTS6 B10 - REAL ANALYSIS

(Mathematics - Core Course)

(2019 Admission - Regular)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 5

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Give an example of a function which is continuous on a set but don't have an absolute maximum on that set.
2. State Preservation of intervals theorem.
3. Show that a Lipschitz function is uniformly continuous.
4. State non uniform continuity criteria.
5. Show that $f(x) = {}^{1}/_{x}$ is not uniformly continuous on $[0, 1]$
6. Define norm of a partition. Find the norm of $\{0, {}^{1}/_{4}, {}^{1}/_{2}, 1, 2\}$ of $[0, 2]$.
7. Find $S(f;\ \dot{P})$ where $f(x) = x^2$ on $[0, 1]$ and $\dot{P}$ is a partition which divides $[0, 1]$ into four equal parts and tags are chosen to be the left end points.
8. State the Fundamental theorem of Calculus (first form).
9. Give an antiderivative of the $signum$ function in $[-5, 5]$.
10. Find $\int_0^2 t^2\sqrt{1+t^3}\,dt$ with proper justifications.
11. Find $\lim\limits_{n\to\infty}\frac{\sin nx}{1+nx}$
12. Test for convergence the series $\frac{1}{3} + \frac{\sqrt{2}}{5} + \frac{\sqrt{3}}{7} + \frac{\sqrt{5}}{9} + \cdots$
13. Define absolutely convergent improper integral with an example.
14. Prove that Beta function is symmetric.
15. Prove that $\Gamma(p+1) = p\,\Gamma(p)$ for all $p > 0$.

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. State and prove boundedness theorem on continuous functions.
17. If $f: [0, 1] \to [0, 1]$ is continuous, then show that $f(x) = x$ for at least one x in $[0, 1]$.
18. Show that if $f \in \mathcal{R}[a, b]$, then $kf \in \mathcal{R}[a, b]$ and $\int_a^b kf = k\int_a^b f$

19. State and prove Squeeze theorem.

20. Test for Uniform convergence, the sequence $\{ e^{-nx}\}$ for $x \geq 0$.

21. State and prove Cauchy criterion for uniform convergence of sequence of functions.

22. Find the Cauchy Principal Value of the improper integral $\int_{-1}^{5} \frac{dx}{(x-1)^3}$

23. Find the relation connecting Beta and Gamma functions.

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. State and prove:

    (a) Location of Roots theorem.

    (b) Bolzano's intermediate value theorem.

25. State and prove Additivity Theorem.

26. Discuss in detail, the convergence of $p-$ series.

27. Prove that, the series $S_n\,(x) = nx(1-x)^n$ can be integrated term by term in $0 \leq x \leq 1$ although it is not uniformly convergent in that interval.

(2 × 10 = 20 Marks)

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