21P202 (Pages: 2) Name: …………………….

Reg. No: …………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022

(CUCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C07 – REAL ANALYSIS II

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define Cantor set and show that it is closed.
2. Prove that a set of outer measure 0 is measurable.
3. Let *A* be the set of irrational numbers in [0,1]. Show that $m^*(A)=0$.
4. A real valued function that is continuous on its measurable domain is measurable. Prove.
5. Let *E* have measure zero. Show that if *f* is a bounded function on *E*, then *f* is measurable and $\int_E f = 0$.
6. Let *f* be a non negative measurable function on *E*. Then $\int_E f = 0$if and only if *f* =0 a.e on E.
7. If the function *f* is Lipschitz on a closed bounded interval [a,b], then it is absolutely continuous on [a,b].
8. Show that a function of bounded variation on the closed bounded interval [a,b] is differentiable almost everywhere on (a,b).

(8 × 1 = 8 Weightage)

Part B

Two questions should be answered from each unit. Each question carries 2 weightage.

Unit I

9. Prove that if a sigma algebra of subsets of **R** contains intervals of the form (a,∞), then it contains all intervals.
10. Prove that the union of countable collection of measurable sets is measurable.
11. State and Prove Borel Cantelli Lemma.

Unit II

12. State Fatou's Lemma and hence obtain Monotone convergence Theorem.
13. State and Prove Beppo Levi's Lemma.
14. State and prove Chebychev's inequality.

Unit III

15. State and prove Jordan's Theorem.
16. Let f be a Lipschitz function on [a,b]. Then prove that f is of bounded variation of [a,b] and $TV(f) \leqslant c.(b-a)$ for some constant c. Also give an example of a continuous function which is not of bounded variation. (TV- Total Variation)
17. State and prove Minkowski's inequality.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and prove Lebesgue Dominated Convergence Theorem.
19. Show that [0,1] has a subset which fails to be measurable.
20. State and Prove Lebesgue's Theorem.
21. (1) Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise almost everywhere on E to the function f. Show that f is measurable.

    (2) State and prove Simple Approximation Theorem

(2 × 5 = 10 Weightage)

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