21P203 (Pages: 2) Name: …..…………………..

Reg. No: …..…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C08 - TOPOLOGY

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART- A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define topology on any non-empty set X.
2. Write true or false. Justify 'Finite union of 2 or more topologies defined on any non-empty set X is again a topology.'
3. Define base for a topology. Can the same topology have more than one base. Justify your claim.
4. Prove that every quotient space of a discrete space is discrete.
5. Prove that the property of being a discrete space is divisible.
6. Prove that every closed, surjective map is a quotient map.
7. Prove that connectedness is preserved under a continuous surjection.
8. Prove the uniqueness of limits of sequences in a Hausdorff space.

(8 × 1 = 8 Weightage)

PART- B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

Unit-1

9. Define semi open interval topology on the set of real numbers. Show that semi open interval topology is stronger than the usual topology on the set of real numbers.
10. Define derived set of a subset of a tojpological space. If , $\bar{A}$ denote the closure of A, then prove that $\bar{A} = A$, for any closed set A
11. Define hereditary property. Prove that metrisability is a hereditary property.

Unit-2

12. Prove that every second countable space is Lindelöf.
13. Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

14. Prove that every quotient space of a locally connected space is locally connected.

Unit – 3

15. Differentiate between T_0 space and T_1 spaces with examples.
16. Prove that all metric spaces are T_4.
17. Prove that every compact Hausdorff space is T_3.

(6 × 2 = 12 Weightage)

PART- C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Prove that the metric topology on R^n is same as the product topology.
19. 1) A subset of R is connected iff it is an interval.

    2) Every closed and bounded interval is compact.
20. Prove that every path connected space is connected. Is the converse true.
21. A be a closed subset of a normal space X and suppose $f\colon A \to [-1,1]$ is a continuous function. Then prove that there exists a continuous function $F\colon X \to [-1,1]$ such that $F(x) = f(x)$ for all $x \in A$.

(2 × 5 = 10 Weightage)

****