21P204 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C09 - ODE & CALCULUS OF VARIATIONS

(Mathematics)

(2019 Admission onwards)

Time : 3 HoursMaximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Find a power series solution of the differential equation. $x^2y'=y$.
2. Verify that $\log(1+x)=xF(1,1,2,-x)$.
3. Find the first three terms of the Legender series of $f(x)=\begin{cases}0 & \text{if } -1\le x<0\\ x & \text{if } 0\le x\le 1\end{cases}$
4. If $W(t)$ is the Wronskian of the two solutions $\begin{cases}x=x_1(t)\\ y=y_1(t)\end{cases}$ and $\begin{cases}x=x_2(t)\\ y=y_2(t)\end{cases}$ of the homogeneous system $\begin{cases}\frac{dx}{dt}=a_1(t)x+b_1(t)y\\ \frac{dy}{dt}=a_2(t)x+b_2(t)y,\end{cases}$ t either identically zero or nowhere zero on $[a,b]$.
5. For the nonlinear system $\frac{dx}{dt}=y(x^2+1), \frac{dy}{dt}=2xy^2$, find the critical points and equation of the paths.
6. Determine the nature and stability properties of the critical point $(0,0)$ of the linear autonomous system $\frac{dx}{dt}=2x, \frac{dy}{dt}=3y$.
7. Using Picard's method of successive approximation, solve the initial value problem $y'=x+y$ with $y(0)=1$ (start with $y_0(x)=1+x$).
8. Show that $f(x,y)=xy^2$ satisfies a Lipschitz condition on any rectangle $a\le x\le b$ and $c\le y\le$

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Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Find the general solution of $y''+xy'+y=0$ in terms of power series in x .
10. Determine the nature of the point $x=\infty$ for the Euler's equation $x^2y''+4xy'+2y=0$
11. Show that $(x-t)\sum_{n=0}^{\infty}P_n(x)t^n=(1-2xt+t^2)\sum_{n=1}^{\infty}nP_n(x)t^{n-1}$

UNIT - II

12. Show that $\frac{d}{dx}J_0(x)=-J_1(x)$ and $\frac{d}{dx}[xJ_1(x)]=xJ_0(x)$

13. Find the critical points and then describe the phase portrait of the system $\frac{dx}{dt}=1, \frac{dy}{dt}=2.$

14. Show that $(0,0)$ is an unstable critical point for the system $\frac{dx}{dt}=2xy+x^3, \frac{dy}{dt}=-x^2+y^5$

UNIT - III

15. State and prove Sturm separation theorem.

16. Find the extremal of the integral $\int_{x_1}^{x_2}\sqrt{1+(y')^2}\,y\,dx$

17. Find the plane curve of fixed perimeter and maximum area.

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Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Find two independent Frobenius series solutions of the differential equation $2x^2y''+x(2x+1)y'-y=0$.

19. State and prove the orthogonality property of Bessel functions.

20. Find the general solution of $\frac{dx}{dt}=4x-2y, \frac{dy}{dt}=5x+2y.$

21. State and prove Sturm comparison theorem. Also show that the eigen functions of the b problem $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right]+\lambda q(x)y=0; y(a)=y(b)=0$ satisfy relation $\int_a^b q y_m(x) y_n(x)\,dx=0$ if $m\neq n.$

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