20U501 (Pages: 2) Name: ……..……….……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-UG)

CC20U MTS5 B05 - ABSTRACT ALGEBRA

(Mathematics – Core Course)

(2020 Admission - Regular)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 4

Section-A

Answer ***all*** questions. Each question carries 2 marks.

1. Write Addition and Multiplication table for $\mathbb{Z}_5$.
2. Prove that $[a]_n = [b]_n$ if and only if $a \equiv b(mod\ n)$.
3. Find all idempotent elements of $\mathbb{Z}_6$.
4. Find all units in $\mathbb{Z}_{15}$.
5. Find all subgroups of S_3.
6. Define Klein four group.
7. Find order of every element in $\mathbb{Z}_6$
8. Find the maximum possible order of an element in S_{10}
9. Define Factor Group with example.
10. State Cayley's Theorem.
11. Define Kernel of a Group Homomorphism.
12. Define Dihedral Group. Write an example.
13. Define Integral Domain. Write an example.
14. Define center of a Group.
15. Define Automorphism of a Group.

(Ceiling: 25 Marks)

Section -B

Answer ***all*** questions. Each question carries 5 marks.

16. State and Prove Euler theorem.
17. Let p be a prime number then show that for any integer a, $a^p \equiv a(mod\ p)$
18. Show that the set $GL_n(\mathbb{R})$ forms a group under matrix multiplication.
19. Prove that every group of prime order is cyclic.
20. State and Prove Lagrange's Theorem.
21. Write subgroup diagram of $\mathbb{Z}_{18}$
22. Prove that set of all even permutations of S_n is a subgroup of S_n .

23. Show that $Auto(\mathbb{Z}) \cong \mathbb{Z}_2$

24. Give addition and multiplication tables for $\mathbb{Z}_2 \oplus \mathbb{Z}_2$.

(Ceiling: 35 Marks)

PART-C (Essay Type)

Answer any ***two*** questions. Each question carries 10 marks.

25. Show that Every permutation in S_n can be written as a product of disjoint cycles

26. Let G be a group and let H be a subgroup of G. Then show that H is a subgroup of G if and only if H is nonempty and $ab^{-1} \in G$ for all $a, b \in G$

27. Show that every subgroup of a cyclic group is cyclic.

28. Show that every finite Integral Domain is a Field.

(2 × 10 = 20 Marks)

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