

20U505

(Pages: 2)

Name:

Reg. No:

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-UG)

CC20U MTS5 B09 – INTRODUCTION TO GEOMETRY AND

THEORY OF EQUATIONS

(Mathematics – Core Course)

(2020 Admission - Regular)

Time: 2 Hours

Maximum: 60 Marks

Credit: 3

PART A

Answer *all* questions. Each question carries 2 marks.

1. Give the focus - directrix definition of a parabola.
2. Determine the equation of the tangent at a point P with parameter t on rectangular hyperbola with parametric equations $x = t, y = \frac{1}{t}$.
3. Classify the non - degenerate conic with the equation
$$x^2 + 8xy + 16y^2 - x + 8y - 12 = 0$$
4. Define an affine transformation. Give an example.
5. Determine the affine transformation which maps (0,0), (1,0) and (0,1) to the points (2,3), (1,6) and (3, -1) respectively.
6. State Fundamental Theorem of affine geometry.
7. Under what conditions $x^n + c^n$ is divisible by $x + c$?
8. By synthetic division find the quotient and remainder when dividing $-x^4 + 7x^3 - 4x^2$ by $x - 3$.
9. State Identity Theorem and Fundamental Theorem of Algebra.
10. Factorize $x^4 + 4$ into real linear and quadratic factors.
11. State Rolle's Theorem.
12. How many real roots do the equation $x^4 + x^2 - x - 3 = 0$ have?

(Ceiling: 20 Marks)

PART B

Answer *all* questions. Each question carries 5 marks.

13. State and prove focal distance property of hyperbola.
14. Prove that set of Euclidean transformations of \mathbf{R}^2 forms a group under the operation of composition of functions.
15. Prove that an affine transformation preserves ratios of lengths along parallel straight lines.

16. Solve the cubic equation $x^3 + 2x^2 + 3x + 2 = 0$ whose roots are a, b and c if $a = b + c$.

17. Solve the equation $x^3 + 9x - 2 = 0$ using Cardan's formula.

18. Solve $x^4 - 8x^2 - 4x + 3 = 0$.

19. Verify that the equation $x^3 - 7x + 7 = 0$ has roots in $(-4, -3)$.

(Ceiling: 30 Marks)

PART C

Answer any *one* questions. The question carries 10 marks.

20. State and prove reflection property of the ellipse.

21. Find the rational roots of the equation $25x^4 - 70x^3 - 126x^2 + 414x - 243 = 0$.

(1 × 10 = 10 Marks)
