22P103 (Pages: 2) Name: ……………….………

Reg. No: ……………..……..

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C03 – REAL ANALYSIS - I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that the neighbourhood of a point in a metric space in an open set.
2. Show that the set of limit points of a bounded set is bounded.
3. Construct a bounded set of real numbers with exactly two limit points.
4. Is arbitrary intersection of closed sets closed? Justify your answer.
5. Prove that every uniformly continuous function in an interval is continuous in that interval. Is the converse true? Justify.
6. Give example for a function which is not Riemann Stieltjes integrable with an example.
7. Define rectifiable curve with an example.
8. Prove that every member of an equi-continuous family of functions is uniformly continuous.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT 1

9. Prove that the interior of a set S is the largest open subset of S.
10. Define Compact set. Explain with details why $S = (0,1)$ is not compact.
11. If F is closed and K is compact, prove that $F \cap K$ is compact.

UNIT 2

12. Let f be defined by $f(x) = \sqrt{x+1}$. Is f uniformly continuous on $\mathcal{R}$. Justify your answer.
13. If $f \in \mathcal{R}(\alpha_1)$ and $f \in \mathcal{R}(\alpha_2)$, Prove that $f \in \mathcal{R}(\alpha_1 + \alpha_2)$ and find $\int_a^b f d(\alpha_1 + \alpha_2)$.
14. State Taylors theorem. Illustrate with an example.

UNIT 3

15. If $\{f_n\}$ is a sequence of continuous functions defined on E in a metric space and if $f_n \to f$ uniformly on E. Prove that f is continuous on E.
16. Let K be a compact metric space and let $f_n \in C(K)$ for $n = 1,2,3$ .... If f_n converges uniformly on K, then prove that $\{f_n\}$ is equi-continuous on K.
17. For $n = 1,2,3$ ...and x real let $f_n(x) = \frac{x}{1+nx^2}$. Show that $\{f_n\}$ converges uniformly.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.
19. State and prove Taylor's theorem.
20. Find a real continuous function on the real line which is nowhere differentiable.
21. State and prove Stone-Weierstrass theorem.

(2 × 5 = 10 Weightage)

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