22P104 (Pages: 2) Reg. No.............................

Name: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C04 - DISCRETE MATHEMATICS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each carries 1 weightage.

1. Define poset. Give example of a poset with and without a maximum element.
2. Show that in a Boolean algebra $(X, +, ., ')$
$$x + (y + z) = (x + y) + z \text{ and } x.(y.z) = (x.y).z \ \forall x, y, z \in X$$
3. Define Characteristic number of a symmetric Boolean function. Find the Characteristic numbers of $f(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_1x_3$
4. Define self complementary graphs. Show that if G is self complementary graph of order n, then $n \equiv 0$ or $1 (\mod 4)$
5. Show that if $\{x, y\}$ is a 2-edge cut of G, then every cycle of G containing x must also contain y.
6. Prove or disprove: If H is a subgraph of G, then
   (a) $\kappa(H) \leq \kappa(G)$
   (b) $\lambda(H) \leq \lambda(G)$
7. Find a grammar that generates $L = \{w \in \{a\}^*; |w| \mod 3 = 0\}$
8. Show that the language $L = \{awa; w \in \{a, b\}^*\}$ is regular.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

Unit I

9. Let $(X, \leq)$ be a poset and A a nonempty finite subset of X. Then A has atleast one minimal element. Also A has a minimum element if and only if it has a unique minimal element.
10. Let $(X, +, ., ')$ be a finite Boolean algebra. Show that every element $x \in X$ can be uniquely expressed as the sum of all atoms contained in x.

11. Define D.N.F and C.N.F. Write the D.N.F and C.N.F of $f(x_1, x_2, x_3) = x_1x_2 + x_2'x_3$

Unit II

12. (a) Show that the connectivity and edge connectivity of a simple cubic graph G are equal

    (b) Draw a simple graph with $\kappa = 1, \lambda = 2,$ and $\delta = 3$.

13. (a) Show that every connected graph contains a spanning tree.

    (b) Draw a spanning tree of K_5 and a spanning sub-graph which is not a tree.

14. (a) State and prove Euler formula for connected plane graphs.

    (b) Define self dual graph and show that for self dual graphs $2n = m + 2$

Unit III

15. Show that $|u^n| = n|u|$ for all strings $u \in \Sigma^*$ and all n

16. (a) Define reverse of a string. Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$

    (b) Prove that $(L_1L_2)^R = L_2^R L_1^R$ for all languages L_1 and L_2.

17. Find a dfa and an nfa that accepts all strings on $\Sigma = \{a, b\}$ starting with the prefix ab.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

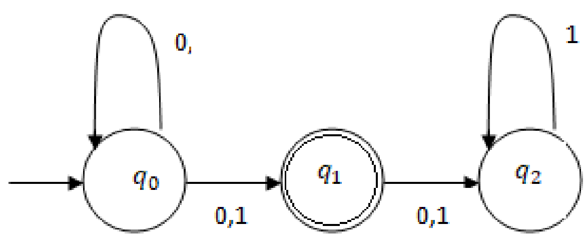
18. State and prove Stone representation theorem for finite Boolean algebras.

19. (a) Show that a graph is bipartite if and only if it contains no odd cycles.

    (b) Prove that every tree is bipartite

20. Show that a graph G with atleast three vertices is 2 connected if and only if any two vertices of G lies on a common cycle.

21. Convert the nfa into equivalent dfa



(2 × 5 = 10 Weightage)

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