22P157 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C04 / CC22P MST1 C04 - PROBABILITY THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Explain independence of events. Show that subclasses of independent classes are independent.

2. What do you mean by probability space? If A_n is a sequence of events and A_n converges to A then show that $P(A_n)$ converges to $P(A)$.

3. Verify whether following functions are distribution function.
(i) $F(x) = \frac{1}{\pi} tan^{-1} x - \infty < x < \infty$
(ii) $F(x) = \begin{cases} 0 & if\, x \leq 1 \\ 1 - \frac{1}{x} & if\, x \geq 1 \end{cases}$

4. Prove that a distribution function can have atmost countable number of discontinuities.

5. State and prove multiplication theorem for two independent non negative random variables.

6. Show that the probability density function of a random variable is symmetric if and only if its characteristic function is real.

7. Let $\{X_n\}$ and $\{Y_n\}$ be two sequences of random variables and If $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{L} c$ then show that $X_n + Y_n \xrightarrow{L} X + c$

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. a) Show that a sigma field is monotone field and conversely
b) Let X be a random variable defined over a probability space (Ω, A, P). Show that $aX + b$ is a random variable.

9. (i) State and prove Jensen's inequality.
(ii) Show that characteristic function of a random variable is non negative definite.

10. State and prove Kolmogorov 0-1 law of probability.

11. Define convergence in probability. If $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$, prove that $X_n Y_n \xrightarrow{P} XY$ as $n \to \infty$.

12. State and prove Levy's continuity theorem.

13. Let F be the distribution function of a random variable X. Then show that F can be decomposed as $F = \alpha F_c + (1-\alpha)F_d$ where F_c is continuous and F_d is a step function and $0 \leq \alpha \leq 1$.

14. State and prove necessary and sufficient condition to hold WLLN's.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. a) Describe any three properties of distribution function

 b) Show that a necessary and sufficient condition for a distribution function to be continuous at a point x is $p(X = x) = 0$

16. Define characteristic function. Check whether |ϕ(t)|| is integrable in the following case, and if so obtain the probability density function using inversion theorem. $\phi(t) = e^{i2t}$

17. a) If X is a random variable taking values 1,2,3,… and $P(X = i) = pi, i = 1, 2, 3...$ then show that $E(X) = \sum_{n=1}^{\infty} P(X \geq n)$

 b) Derive Basic Inequality

18. (a) State and prove Helly Bray theorem.

 (b) State and prove Lindeberg-Levy's form of central limit theorem.

(2 × 5 = 10 Weightage)

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