22P155 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C02 / CC22P MST1 C02 - ANALYTICAL TOOLS FOR STATISTICS – II

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any **four** questions. Each question carries 2 weightage.

1. Let V be a finite dimensional vectorspace and let (v_1, v_2,...v_n) be any basis. Show that

   (i) If a set has more than n vectors; then it is linearly dependent.

   (ii) If a set has fewer than n vectors; it does not span V

2. if W_1 and W_2 are finite subspace of a vectorspace V then prove that $d(W_1 \cap W_2) = r + m + n$,

3. Define skew hermitian matrix and its properties.

4. Reduce the following matrix in to normal form and find its rank

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

5. Define characteristic and minimal polynomial. Show that minimal polynomial of a matrix A exists and it is unique.

6. State and prove the necessary and sufficient condition for a system Ax = B is consistant.

7. Prove that every quadratic form can be reduced to a form containing square terms by a non-singular linear transformation.

(4 × 2 = 8 Weightage)

Part-B

Answer any **four** questions. Each question carries 3 weightage.

8. Write a short note on Gram- Schmidt orthogonalization process.

9. Define orthogonal matrix. Let A be an arbitrary 2 x 2 (real) orthogonal matrix, prove : If (a, b) is the first row of A, then $a^2 + b^2 = 1$ and A= $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ or A= $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.

10. Find rank and basis of the row space of the following matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$

11. Show that geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.

12. Find the spectral decomposition of $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

13. Define g-inverse . Find g-inverse of A= $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$.

14. Classify the following the quardratic form positive definite, positive semi definite and indefinite $12x^2 + 2y^2 + 6z^2 - 4yz - 4zx + 2xy$.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. (a) State and prove rank nullity theorem.

(b) Reduce the matrix in to normal form and find rank. $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$.

16. (a) Explain spectral decomposition of a matrix.

(b) Find spectral decomposition of the matrix A= $\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$.

17. (a) State and prove necessary and sufficient condition for the diagonalizability of a square matrix.

(b) If A is a hermitian matrix, show that A is unitarily similar to a diagonal matrix.

18. (a) State and prove the necessary and sufficient condition that a real quadratic form X'AX is positive definite.

(b) Classify the quadratic form $x_1^2 + 4x_2^2 + x_3^2 - 4x_2x_3 + 2x_3x_1 - 4x_1x_2$.

(2 × 5 = 10 Weightage)

****