22P156 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C03 / CC22P MST1 C03 - DISTRIBUTION THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. If X and Y are independent binomial random variable such that X~ B(m, p), Y~ B(n, p). Show that X|(X+ Y) is hyper geometric.
2. Derive the recurrence relation for cumulants of generalised Power series distribution.
3. Define Weibull distribution. Obtain the distribution of U where $U = Min(X_1, X_2, ..., X_n)$ if $X_i's$ independently distributed according to standard Weibull.
4. Define Cauchy Distribution and obtain its quartile deviation.
5. Derive the pdf of Pearson type II distribution.
6. $X_1, X_2, ..., X_n$ are independently and identically distributed geometric random variables with parameter p. Obtain the distribution of $X_{(1)} = \min(X_1, X_2, ..., X_n)$.
7. Let X ~$N(0,1)$ and Y ~ $\chi^2_{(n)}$ and X and Y are independent. Obtain the distribution of $\frac{X}{\sqrt{\frac{Y}{n}}}$.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Define m.g.f of a random variable. Find the mgf of i) $Y = aX + b$ ii) $Y = \frac{X-m}{\sigma}$
9. Let X and Y be independent random variables following the negative binomial distributions, $NB(r_1, p)$ and $NB(r_2, p)$ respectively. Show that the conditional probability mass function of X given $X + Y = t$ is hypergeometric.
10. The probability density function of a random variable X is given by $f(x) = \frac{k a^k}{x^{k+1}}$, $x \geq a$, k >0. Derive the geometric mean of X
11. Obtain the moment generating function of Gamma distribution. Establish the additive property of Gamma distribution.

12. Let $f(x, y) = 8xy,\ 0 < x < y < 1$ be the joint probability density function of X and Y. Find E(Y|X).
$= 0$, otherwise

13. If X and Y are independent one parameter gamma variates with parameters m and n Derive the joint distribution of the following and hence marginal distributions $U = X + Y \; and \; V = \frac{X}{X+Y}$

14. In sampling from a normal population, show that the sample mean $\bar{X}$ and the sample variance S^2 are independently distributed.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Define probability generating function. If P(s) is the probability generating function for the random variable X, find the probability generating function for a) $\frac{(X-a)}{b}$ b) X+1 c) 2X

16. a) Show that the exponential distribution 'lacks memory".
b) If $X_1, X_2, \ldots, X_n$ are independent exponential random variables with parameter $\theta_i; i = 1, 2, \ldots, n$. Obtain the distribution of $Z = min(X_1, X_2, \ldots, X_n)$

17. If X and Y follow bivariate normal distribution. Find (i) the conditional distribution of X given Y (ii) the marginal distributions of X and Y.

18. Define non-central χ^2 distribution. Obtain its moment generating function.

(2 × 5 = 10 Weightage)

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