21P302 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C12 - COMPLEX ANALYSIS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Consider the stereographic projection between $\mathbb{C}_\infty$ and $S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$. Let $z = x + iy \in \mathbb{C}$ and $Z = (x_1, x_2, x_3)$ be the corresponding point of S. Express $Z = (x_1, x_2, x_3)$ in terms of z.
2. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a^n z^n;\ a \in \mathbb{C}$.
3. State and prove symmetry principle.
4. Evaluate $\int_\gamma \frac{dz}{z-a}$ where $\gamma(t) = a + re^{it},\ 0 \le t \le 2\pi$.
5. State and prove fundamental theorem of algebra.
6. Prove that $f(z) = \frac{\sin z}{z}$ has a removable singularity at $z = 0$. Define $f(0)$ so that f is analytic at $z = 0$.
7. For $|a| < 1$, let $\varphi_a(z) = \frac{z-a}{1-\bar{a}z}$. Prove that the inverse of φ_a is φ_{-a}.
8. Hadamard three cycle theorem.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. If G is open and connected and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0,\ \forall z \in G$, prove that f is constant.
10. If $Tz = \frac{az+b}{cz+d}$, find z_2, z_3, z_4 in terms of a, b, c, d such that $Tz = (z, z_2, z_3, z_4)$.

11. Let $\gamma : [a, b] \to \mathbb{C}$ is a rectifiable path and $\varphi : [c, d] \to [a, b]$ is a continuous non-decreasing function with $\varphi(c) = a$ and $\varphi(d) = b$. Prove that for any function *f* continuous on $\{\gamma\}$, $\int_{\gamma} f = \int_{\gamma \circ \varphi} f$.

UNIT - II

12. Let *f* be analytic in $B(a; R)$. Prove that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ for $|z - a| < R$ where $a_n = \frac{1}{n!} f^{(n)}(a)$ and this series has radius of convergence greater than or equal to ***R***.

13. Let G be a region and $f : G \to \mathbb{C}$ be a continuous function such that $\int_{\gamma} f = 0$ for every triangular path T in G. Prove that *f* is analytic in G.

14. Prove that if G is simply connected and $f : G \to \mathbb{C}$ is analytic in G, then *f* has a primitive in G.

UNIT - III

15. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^4} dx$.

16. State and prove argument principle.

17. State and prove maximum modulus principle (any version).

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let *u* and *v* be real valued functions defined on a region G and suppose that *u* and *v* have continuous partial derivatives. Prove that $f : G \to \mathbb{C}$ defined by $f(z) = u + iv$ is analytic iff *u* and *v* satisfy the Cauchy-Riemann equations.

19. Let z_1, z_2, z_3, z_4 be four distinct points in $\mathbb{C}_\infty$. Prove that (z_1, z_2, z_3, z_4) is a real number iff all four points lie on a circle. Then prove that every Mobius transformation maps circles onto circles.

20. Let G be an open set and $f : G \to \mathbb{C}$ be a differentiable function. Prove that *f* is analytic on G.

21. Evaluate $\int_0^{\pi} \frac{d\theta}{(a + \cos\theta)^2}$ where $a > 1$.

(2 × 5 = 10 Weightage)

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