21P303 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C13 - FUNCTIONAL ANALYSIS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Prove that any two cosets of a linear space are either disjoint or identical.
2. Prove that if f_i is a complete system in a Hilbert space H and $x \perp f_i$, then $x = 0$.
3. Define projection of x in H onto L, where L is a closed subspace of H.
4. Show that for every closed subspace of H,$L \oplus L^{\perp} = H$
5. State Hahn-Banach Theorem. Show that for all $x_1 \in X,$ and for all $x_2 \in X$ such that $x_1 \neq x_2$ there exists $f \in X^*$ satisfying $f(x_1) \neq f(x_2)$.
6. Prove that for a bounded linear operator A if the image of a unit ball is precompact then the image of any ball is precompact
7. Define Norm convergence and strong convergence in $L(X)$.
8. If A and B are invertible opertaors then prove that AB is ivertible and $(AB)^{-1} = B^{-1}A^{-1}$

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Let O be an openset then prove that $F = O^c$ closed . Also prove if F is closed set then F^c is open.
10. Let X_0 be a closed subspace of X.. Verify X/X_0 is a normed space together with the norm defined by $||[x]|| = \inf_{y \in X_0} ||x - y||$
11. Let E be a normed space. Prove that there exists a complete normed space $E^1$and a linear operator $T : E \to E^1$ such that $||Tx|| = ||x||$ for all $x \in E$. Also prove image T is dense in E^1.

UNIT - II

12. State and prove Parallelogram law in Hilbert Space

13. State and Prove Bessel's inequality

14. Consider $f \in E^{\#} - \{0\}$. Then prove that

    1. codim ker $f = 1$
    2. If $f, g \in E^{\#} - \{0\}$ and ker $f =$ ker g, then prove that there exists $\lambda \neq 0$ such that $\lambda f = g$.

UNIT - III

15. Let $L \hookrightarrow X$ be a subspace of a normed space X and let $x \in X$ such that dist $(x, L) = d > 0$. Then, prove that there exists $f \in X^*$ such that $\|f\| = 1, f(L) = 0$ and $f(x) = d$.

16. Prove that $K(X \to Y)$ is a closed linear subspace of $L(X \to Y)$.

17. If $A : X \to Y$ is compact then prove that $A^* : Y^* \to X^*$ is compact.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and prove Holder's inequlaity for sequences

19. Show that the Hilbertspace is seperable if and only if there exist a complete orthonormal system $\{e_i\}_{i \geq 1}$

20. State and prove Riesz representation theorem.

21. Let X be a normed space and let Y be a complete normed space. Then prove that $L(X \to Y)$is a Banach Space.

(2 × 5 = 10 Weightage)

****