21P362 (Pages: 2) Name: ……………………..

Reg. No: …………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P ST3 C12 - STOCHASTIC PROCESSES

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define stochastic processes and Markov processes with an example.
2. State and prove Chapman – Kolmogorov equations.
3. Show that the interarrival time of Poisson process follows exponential distribution.
4. Explain compound Poisson process and conditional or mixed Poisson process.
5. Show that renewal function satisfies renewal equation.
6. What is Little's formula and explain Kendall's Notation?
7. Define weakly stationary and strictly stationary process.

(4 × 2 = 8 Weightage)

PART B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, and 2 with transition probability matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution $P[X_0 = i] = 1/3$, $i = 0, 1, 2$

   Find (a) $P[x_3 = 1, x_2 = 2, x_1 = 1, x_0 = 2]$ (b) $P[x_3 = 1]$
9. Obtain mean and variance of a continuous time Branching process.
10. Explain linear growth model with immigration.
11. State and prove elementary renewal theorem
12. Explain Alternating renewal process and semi Markov process
13. Define queue with an example. What are the characteristics of queue?
14. Obtain Pollaczek – Khinchin formula.

(4 × 3 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

15\. a) Let X be a Markov chain with state space E = {1, 2, 3} and transition probability matrix $\begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.6 & 0 & 0.4 \\ 0 & 0.4 & 0.6 \end{bmatrix}$. Find limiting probabilities

b) Explain Gambler's ruin problem.

16\. a) Show that the sum of two independent Poisson process is a Poisson process and difference of two independent Poisson process is not a Poisson process.

b) Show that a Poisson process is a Markov process

17\. a) Describe insurance ruin problem.

b) What is inspection paradox?

18\. a) Describe $M|M|1$ queue. Find expected number of customers in the system and the queue and expected waiting time in the system and the queue

b) Explain Brownian motion.

(2 × 5 = 10 Weightage)

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