22U202 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS2 C02 / CC20U MTS2 C02 - MATHEMATICS - II

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Convert $(\sqrt{3}, 1)$ from Cartesian coordinates to polar coordinates
2. If $f(x) = x^5 + x^3 + 2x$, find derivative of the inverse function, $g'(0)$.
3. Replace the Cartesian equation $y^2 = 4x$ by the equivalent polar equation.
4. Evaluate $\displaystyle\int \frac{\sinh x}{\cosh^4 x} dx$.
5. Express $\tanh^{-1}\left(\frac{1}{2}\right)$ in terms of natural logarithms.
6. Find $\lim_{n\to\infty} \left(\dfrac{(-1)^n + n}{n}\right)$.
7. Sum the series $\displaystyle\sum_{i=0}^{\infty} \frac{3^i - 2^i}{6^i}$
8. Test for convergence of the series $\displaystyle\sum_{i=1}^{\infty} \frac{1}{3i + 1/i}$
9. Show that $\displaystyle\sum_{i=1}^{\infty} \frac{(-1)^i}{i\ 3^{i+1}}$ converges.
10. Define basis of a vectorspace. Write the standard basis of R^4.
11. If $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 5$. Evaluate $\det A$ where $A = \begin{bmatrix} a_1 - 2b_1 + 3c_1 & a_2 - 2b_2 + 3c_2 & a_3 - 2b_3 + 3c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$
12. Determine whether the matrix $\begin{pmatrix} \frac{1}{3} & -1 \\ 4 & 3 \end{pmatrix}$ is singular or nonsingular.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Find the area of the surface obtained by revolving the graph of the function $y = x^2,\ 1 \le x \le 2$, about the y-axis.

14. For what values of r $\int_0^1 x^r\ dx$ is convergent?

15. Evaluate $\int_0^{\frac{\pi}{2}} \cos x\ dx$ using Simpson's rule with $n = 10$. Compare the answer with the true value.

16. The set $B = \{u_1, u_2, u_3\}$ where $u_1 = \langle 1,1,1\rangle,\ u_2 = \langle 1,2,2\rangle,\ u_3 = \langle 1,1,0\rangle$ is a basis of R^3. Transform B into an orthonormal basis.

17. Find the rank of the matrix $\begin{bmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 1 & 0 & 5 & 1 \\ 2 & 1 & \frac{2}{3} & 3 & \frac{1}{3} \\ 6 & 6 & 6 & 12 & 0 \end{bmatrix}$

18. Find the eigen values and the eigen vector corresponding to any one of the eigen value of the matrix $A = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$

19. If $A = \begin{bmatrix} -2 & 4 \\ -1 & 3 \end{bmatrix}$, prove that $A^6 = 22I + 21A = \begin{bmatrix} -20 & 84 \\ -21 & 85 \end{bmatrix}$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. Balance the chemical equation $C_2H_6 + O_2 \to CO_2 + H_2O$.

21. Find an orthogonal matrix P that diagonalizes the symmetric matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Also find the diagonal matrix D such that $D = P^T AP$.

(1 × 10 = 10 Marks)

****