21U401 (Pages: 2) Name: ....

Reg.No: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS4 B04 / CC20U MTS4 B04 - LINEAR ALGEBRA

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find all values of k for which the given augmented matrix corresponds to a consistent linear system $\begin{bmatrix} 1 & k & -4 \\ 4 & 8 & 2 \end{bmatrix}$
2. When a homogeneous linear system posses infintely many solutions?
3. Solve $3x - 2y = -1;\ 4x + 5y = 3$, by using matrix inversion.
4. Find the Values of λ for which $det(A) = 0$, when $A = \begin{bmatrix} \lambda - 1 & 0 \\ 2 & \lambda + 1 \end{bmatrix}$
5. If A is a square matrix which has a row of zeros or a column of zeros then prove that $det A = 0$
6. If $u = (1, 2, -1)$ and $v = (6, 4, 2)$, show that $w = (9, 2, 7)$ is a linear combination of u and v
7. Check whether $f(x) = x$ and $g(x) = sinx$ are linearly independent or not.
8. Using plus-minus theorem show that $p_1 = 1 - x^2, p_2 = 2 - x^2\ and\ p_3 = x^3$ are linearly independent
9. Define Column space.
10. Use matrix multiplication to find the reflection of $(2, -5, 3)$ about xy- plane
11. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $w_1 = 2x_1 + x_2$ and $w_2 = 3x_1 + 4x_2$. Check whether $T(1, 1) = T(1, 0) + T(0, 1)$
12. Find the equation of the image of the line $y = 2x + 1$ under the multiplication by matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$
13. Check whether $f(x) = x\ and\ g(x) = x^2$ are orthogonal with respect to $\langle f, g \rangle = \int_a^b f(x).\, g(x)$
14. Define an orthonormal set
15. When we say that the matrix B is orthogonally similar to A

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Show that if a square matrix A satisfying the equation $A^2 + 2A + I = 0$, then A must be invertible.What is the inverse?

17. Let A and B are square matrices of the same size.If AB is invertible then prove that A and B are invertible.

18. Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be multiplication by $A = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 2 \\ 4 & 5 & -3 \end{bmatrix}$ and let e_1, e_2 and e_3 be the standard basis vectors for $\mathbb{R}^3$. Find (a) $T_A(e_1), T_A(e_2), T_A(e_3)$ (b) $T_A(e_1 + e_2 + e_3)$ (c) $T_A(7e_3)$

19. Given a basis $S = \{v_1, v_2, v_3\}$ of $\mathbb{R}^3$,where $v_1 = (1,2,1), v_2 = (2,9,0), v_3 = (3,3,4)$ find the coordinate vector of $V = (5,-1,9)$ relative to this basis

20. Consider the bases $B = \{u_1, u_2\}$ and $B' = \{u_1', u_2'\}$ where $u_1 = (2,2), u_2 = (4,-1), u_1' = (1,3), u_2' = (-1,1)$ find the transition matrix from B to B'

21. Find the eigen values of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

22. Show that $\langle f, g \rangle = \int_a^b f(x).\, g(x)$ is an inner product space on $C[a,b]$

23. Show that $A = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$ is orthogonal and find its inverse.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Solve the matrix equation for$X$, if $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$

25. Show that the spaces R^2 *and* R^3 are vector spaces

26. a) State and prove Dimension Theorem

    b) Verify Dimension Theorem for $A = \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

27. Let A be an $n \times n$ matrix. Prove that A is diagonalizable if and only if A has n linearly independent vectors.

(2 × 10 = 20 Marks)

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