20U602 (Pages: 2) Name: ....

Reg.No: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - UG)

CC20U MTS6 B11 - COMPLEX ANALYSIS

(Mathematics - Core Course)

(2020 Admission - Regular)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Evaluate $\lim\limits_{z \to 1-i} \left(|z|^2 - i\bar{z} \right)$
2. Show that the function $f(z) = \begin{cases} \dfrac{z^3 - 1}{z - 1} & \text{if } z \neq 1 \\ 3 & \text{if } z = 1 \end{cases}$ is continuous at $z = 1$.
3. Show that $f(z) = x + 4iy$ is nowhere differentiable.
4. Show that $f(z) = \text{Re}(z)$ is nowhere analytic.
5. Express the function $f(z) = e^{1/z}$ in the form $f(z) = u(x, y) + iv(x, y)$.
6. Express the value of $\tan(\pi - 2)i$ in the form $a + ib$.
7. Show that $\cosh(-z) = \cosh z$.
8. Evaluate $\oint_C x dx$ where C is the circle defined by $x = \cos t, y = \sin t;\ 0 \leq t \leq 2\pi$.
9. Evaluate $\oint_C \tan z \, dz$ where C is the circle $|z| = 1$.
10. State Cauchy's intgral formula for derivatives.
11. Prove that the only bounded entire functions are constants.
12. Determine whether the sequence $\{1 + i^n\}$ converges or diverges.
13. Determine whether the geometric series $\sum\limits_{k=0}^{\infty} (1 - i)^k$ convergent or divergent.
14. Determine the zeros and their order for the function $f(z) = z^4 + z^2$
15. Let $f(z) = \dfrac{1}{(z-1)^2(z-3)}$, find $\text{Res}(f(z), 3)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Show that the function $f(z) = \dfrac{x-1}{(x-1)^2+y^2} - i\dfrac{y}{(x-1)^2+y^2}$ is analytic in an appropriate domain. Also find $f'(z)$ in that domain.

17. Find all complex values of z satisfying the equation $e^{1/z} = -1$.

18. Evaluate $\int_C (x^2 + iy^3)dz$ where C is the straight line from $z = 1$ to $z = i$.

19. Evaluate $\int_i^{1+i} ze^z dz$

20. State Cauchy's integral formula. Using Cauchy's integral formula evaluate $\oint_C \dfrac{z^2+4}{z^2-5iz-4}dz$ where C is the circle $|z-3i| = 1.3$.

21. Find the circle and radius of convergence of the power series $\sum_{k=0}^{\infty} \dfrac{(z-4-3i)^k}{5^{2k}}$

22. Expand $f(z) = \dfrac{1}{3-z}$ in a Taylor series with center $z_0 = 2i$. Give the radius of convergence R.

23. Evaluate $\int_0^{\pi} \dfrac{1}{2-\cos\theta}d\theta$.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Show that the function $u(x, y) = \log_e(x^2 + y^2)$ is harmonic in an appropriate domain. Also find the harmonic conjugate function of u.

25. State and prove the ML inequality. Use it to find an upper bound for the absolute value of the integral $\int_C \dfrac{e^z}{z^2+1}dz$ where C is the circle $|z| = 5$.

26. Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in a Laurent series valid for the following annular domains

    1. $1 < |z| < 2$ 2. $|z| > 2$ 3. $0 < |z-1| < 1$ 4. $0 < |z-2| < 1$

27. State residue theorem. Using residue theorem evaluate $\oint_C \dfrac{z+1}{z^2(z-2i)}dz$ where C is the circle.

    1. $|z| = 1$ 2. $|z-2i| = 1$ 3. $|z-2i| = 4$

(2 × 10 = 20 Marks)

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