22P201 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C06 - ALGEBRA- II

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that a field F is either of prime characteristic p and contains a subfield isomorphic to $\mathbb{Z}_p$ or of characteristic 0 and contains a subfield isomorphic to $\mathbb{Q}$.
2. Prove that $\mathbb{C}$ is a simple extension of $\mathbb{R}$.
3. Show that a regular 9-gon is not constructible.
4. Prove that finite extension of a finite field is finite.
5. Prove that any two algebraic closures of a field F are isomorphic.
6. Prove that $\mathbb{C}$ is a splitting field over $\mathbb{R}$.
7. State Primitive Element Theorem.
8. Show that the polynomial $x^5 - 2$ is solvable by radicals over $\mathbb{Q}$.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. A finite extension field E of a field F is an algebraic extension of F . What about the converse?
10. Find a basis and dimension of for $\mathbb{Q}(\sqrt{3}, \sqrt{7})$ over $\mathbb{Q}$.
11. State and Prove Fundamental Theorem of Algebra

UNIT - II

12. If F is a field of prime characteristic p, then prove that $(\alpha + \beta)^{p^n} = \alpha^{p^n} + \beta^{p^n}$ for all $\alpha, \beta \in F$ and all positive integers n.
13. If $E \leq \overline{F}$ is a splitting field over F, then prove that every irreducible polynomial in $F[x]$ having a zero in E splits in E.

14. If E is a finite extension of F, then show that E is separable over F if and only if each α in E is separable over F.

UNIT - III

15. Describe the Galois group $G(\mathbb{Q}(\sqrt{2})/\mathbb{Q})$.

16. State Main Theorem of Galois Theory

17. Find $\Phi_3(x)$ over $\mathbb{Z}_2$.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let R be a commutative ring with unity. Then show that M is a maximal ideal of R if and only if R/M is a field.

19. Let F be a field and let $f(x)$ be a nonconstant polynomial in $F(x)$ .Show that then there exists an extension field E of F and an $\alpha \in E$ such that $f(\alpha) = 0$.

20. State and Prove The Conjugation Isomorphism theorem.

21. Let F be a field of characteristic 0 and let $a \in F$. If K is the splitting field of $x^n - a$ over F, then prove that $G(K/F)$is a solvable group.

(2 × 5 = 10 Weightage)

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